2

Mathematical Formulation of the Maximum Em-Power Principle

Corrado Giannantoni

ABSTRACT

After having briefly recalled the main results shown in the previous Conference (a rigorous definition of Emergy in mathematical terms and a general formulation of the Emergy Balance equation) the paper presents an extremely general mathematical formulation of the Maximum Em-Power Principle. Such a formulation allows us to analyze in particular (but not exclusively): i) in what sense the "maximum" flow has to be understood, not only in steady state conditions but also in stationary and variable conditions; ii) what reciprocal role Transformity and Exergy play in maximizing such a flow; iii) in what sense such an extremely General Principle can be interpreted (but only reductively) as a traditional Thermodynamic Principle.

The last aspect enables us to point out the different and wider presuppositions of the M. Em-P. Principle (as a more general "Thermodynamic" Principle) in comparison with the other Thermodynamic Principles (especially the First and the Second ones) and furnishes a clear answer to the (apparently) contradictory assertions between the Maximum Em-Power Principle and the Minimum Action Principle. In addition, such a mathematical formulation throws new light on the thermodynamic concepts of order and disorder, and paves the way to a better understanding of the so-called Fifth Principle.

1. INTRODUCTION

This paper expressly deals with the mathematical formulation of the Maximum Em-Power Principle. To this purpose it is worth starting from the basic results already achieved and presented at the First Emergy Research Conference (Giannantoni, 2000a), which constitute the most correct presuppositions obtained for such a possible formulation. Let us recall two of them very synthetically: a general mathematical definition of Emergy and the structure of the global Emergy Balance Equation.

i) A rigorous and *general definition of Emergy*, valid in whatever variable conditions, can be given by the following expression

$$Em^*(t) = \int_{-\infty}^{t} \dot{Ex}_{eq}(\tau)d\tau \tag{1.1}$$

where $\dot{Ex}_{eq}(\tau)$ is defined as

$$\dot{Ex}_{eq}(\tau) = \int_{D^*(\tau)} c(x,y,z,\tau)\cdot\rho(x,y,z,\tau)\cdot ex(x,y,z,\tau)d_3V \tag{1.2}$$

and is the instantaneous *equivalent* Exergy Power used up during the process of generating a specific product.

Eqs. (1.1) and (1.2) translate, in mathematical terms, the general definition of Emergy given by H. T. Odum as ´the total solar equivalent*available Energy* directly and indirectly used up to generate a specific form of Energy (or product)[a]. In fact, the coefficient which appears in Eq. (1.2) is a *dimensional* structural factor (whose dimensions are sej/J, that is solar emergy joules per Joule) and depends, among other things, on co-injection or co-production factors. It is thus defined in such a way as to *summarize all the rules of Emergetic Algebra* (this is the reason for the term "equivalent"); $D^*(\tau)$ is the Domain of integration which defines the quantity of the considered matter, is the mass density, is the specific Exergy, while Newton's "dot" notation in Eq. (1.1) stands for the *total derivative* with respect to time.

Therefore, when we assume a Lagrangian perspective and steady state conditions, Eqs. (1.1) and (1.2) define the traditional and usual concept of Emergy, while under *whatever variable conditions* they define Emergy in its widest and most general conception.

ii) A general *Global Balance Equation* for any System made up of n sub-systems can be written as follows

$$\sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m\left(u_j\right) + \sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*\left(u_1, u_2, .., u_m\right) = \frac{\partial}{\partial t} A_{D_s}(t) + \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}m\left(y_l\right) \qquad (1.3)$$

where $\left[\frac{sej}{\sec}\right] \Phi_k^*\left(u_1, u_2, .., u_m\right)$ is the "*equivalent*" Source Term1 relative to the *k*- sub-System, which is expressed as a linear combination of all the *real* sub-System contributions

$$\Phi_k^*\left(u_1, u_2, .., u_m\right) = \sum_{r=1}^{n} \lambda_{kr}^* \cdot \Phi_r \qquad (1.4),$$

while $A_{D_s(t)}$ is the *Global Accumulation Term* due to all the distinct contributions of the n sub-systems, thus it is given by an *appropriate sum* of *equivalent* accumulation terms, each one (in turn) expressed as a linear combination of all the *real* sub-System contributions

$$A_{D_s(t)} = \sum_{i=1}^{n} \delta_i^* \cdot \delta_i \cdot A_{D_i}^*\left(A_{D_1}, A_{D_2}, .., A_{D_n}\right) = \sum_{i=1}^{n} \delta_i^* \cdot \delta_i \cdot \sum_{j=1}^{n} \varepsilon_{ij}^* \cdot A_{D_j} \qquad (1.5)$$

where

, are the *co-injection* and *co-production* coefficients for each sub-System

are their associated re-normalization factors (referred to the Whole System)

$, \delta_i$ are the pertinent "weights" in the corresponding sub-system balance equation

$, \delta_i^*$ are the associated re-normalization factors

$\lambda_{kr}^*, \varepsilon_{ij}^*$ are the specific incidence coefficients.

Moreover, it is worth recalling that the considered Global Balance Equation (1.3), in addition to accounting for input and output quantities as a global result of the different *co-injective or co-productive* sub-System structures, contemporarily accounts for three distinct additional contributions:

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Accumulation Terms, amplified by both the *productive* and *connective* structure of the System;

- *Source Terms*, characterized by similar (*productive* and *connective*) amplification effects;

- *Circulating Emergy Flow*, which quantifies the Flow of Information through feed-back pathways.

The latter quantity, if Eq. (1.3) is already structured in its *standard form2*, may be defined as

$$\dot{E}m_{circ} = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}m(y_l) - \sum_{l=1}^{p} \dot{E}m(y_l) \tag{1.6}$$

and corresponds to the'*Flow of Information* through the *feed-back pathways* of the System.

On the basis of such presuppositions we will firstly try to answer a fundamental question.

2. WHAT EXACTLY IS MAXIMUM?

This is certainly not a trivial question. In fact the general *Global Balance Equation* (1.3), which is valid for any Complex System, may be written in several forms by organizing the different terms in such a way as they could appear, according to specific exigencies, either on the first or on the second side of the equation. Let us consider some of these forms.

In addition to Eq. (1.3) which, restructured as follows through Eq. (1.6), may be denominated as form A

A)

$$\sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) + \sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) = \frac{\partial}{\partial t} A_{D_s}(t) + \dot{E}m_{circ} + \sum_{l=1}^{p} \dot{E}m(y_l) \tag{2.2}$$

we could also consider other main forms, for instance the following ones:

B)

$$\sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) = \frac{\partial}{\partial t} A_{D_s}(t) + \dot{E}m_{circ} + \sum_{l=1}^{p} \dot{E}m(y_l) - \sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) \tag{2.3}$$

C)

$$\sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) + \sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) - \sum_{l=1}^{p} \dot{E}m(y_l) = \frac{\partial}{\partial t} A_{D_s}(t) + \dot{E}m_{circ} \tag{2.4}$$

D)

$$\sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) + \sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) - \sum_{l=1}^{p} \dot{E}m(y_l) - \dot{E}m_{circ} = \frac{\partial}{\partial t} A_{D_s}(t) \tag{2.5}$$

which do not evidently exhaust all the various other possibilities.

If we now consider the form A as the *reference structure* in order to formulate the M. Em-P. P., this assumption is equivalent to saying that "the System tends to maximize the sum of input and autogenerated Emergy flows (first side) in order to maximize the rate of accumulated Emergy together with circulating and output Emergy flows (second side)".

Analogously, if we take the other considered forms as reference structure, the pertaining formulation of the M. Em-P. P. would respectively assert that:

B) "The System tends to maximize the autogenerated Emergy flow (first side) in order to maximize the rate of accumulated Emergy, together with circulating and output Emergy flows, at net of the equivalent input one (second side)";

C) "The System tends to maximize the autogenerated Emergy plus the net input/output Emergy flows (first side) in order to maximize the rate of accumulated and circulating Emergy in the system (second side)";

D) "The System tends to maximize the autogenerated Emergy plus the net input/output Emergy flows at net of circulating Emergy flow (first side), in order to maximize the rate of accumulated Emergy in the system (second side)".

Such different possible statements allow us to reformulate the initial question in a more correct form: what is the mathematical structure that is potentially more suitable to best translate Odum's formulation of the Principle under consideration?

We will now try to show that form B is the most suitable starting structure in order to formulate such a *general reference principle* for self-organizing systems which is known as Maximum Em-Power Principle.

3. USEFUL EMERGY AND PROCESSED EMERGY

In many publications H.T. Odum repeatedly asserts that "Em-power" has to be understood as "*useful*

Emergy power". This could lead us to think (erroneously) that the Emergy Balance in form A is the one which is more suitable to represent such a concept, especially if we consider the terms which appear on the second side of Eq. (2.2). But it is also evident that, if the System has all the Source Terms $\Phi_k^*(u_1, u_2, .., u_m)$ equal to zero, it is intrinsically unable to organize input resources, so that it simply transforms the equivalent input Emergy flow into other forms of flow, without any incremental Emergy contribution that might be really *useful* to improve *either* its internal structure *or* its external Emergy products. It is not a self-organizing system at all, but it is only a mechanical *transducer*. In fact, under such conditions, all the pertinent co-production coefficients (β_l) are all equal to 1, as well as the corresponding re-normalization factors (β_l^*). Thus Eq. (1.6) implies that

$$\dot{E}m_{circ} = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}m(y_l) - \sum_{l=1}^{p} \dot{E}m(y_l) = 0 \qquad (3.1).$$

Consequently: a) In *steady state* conditions such a system merely transfers input Emergy flow directly into output; b) In *variable conditions* input Emergy flow is simply transformed into the rate of Accumulated Emergy and output Emergy flow, but without showing any organizing phenomenon (it acts like a mass anchored to a spring and contemporarily subjected to an external forcing energy source). In addition, if the output flow is persistently equal to zero, the system reduces to an Emergy storage (which evidently cannot be defined as a self-organizing system).

We may thus conclude that a *self-organizing* System (in its most meaningful sense) is only the one that "manages" input resources by increasing them by an additional positive and *net* contribution which is the only really *useful* one to *increase* both its rate of Accumulated and Circulating Emergy flow and its output Emergy flows.

In other words, what is really "useful" is not simply an input/output Emergy transfer, but the *net* contribution given by the internal generation process of Emergy which, through an appropriate organization of input resources, finalizes everything to a *net increase* of Emergy both in terms of rate of Accumulation, Circulation and output Production.

It should now be clear that the Balance Equation in form B is the one which is specifically indicated to express Odum's findings. At the same time, if we take into consideration that, apart from the

Emergy Source Term, all the other forms of Emergy are involved in such a (*re*)*organization* process, we may consequently name them by a comprehensive term such as "processed" Emergy flows (obviously each flow is understood as accounted for with its own specific algebraic sign).

We can now propose the researched formulation of the M. Em-P. Principle.

4. MATHEMATICAL FORMULATION OF THE MAXIMUM EM-POWER PRINCIPLE

Eq. (2.3) can be at first re-organized in a more synthetic and compact form. In fact such characteristics, as already pointed out in Giannantoni (2000a), are particularly necessary in order to clearly answer the question as to whether the Maximum Em-Power Principle is really a Thermodynamic Principle or not. At the same time it is also desirable for such a formulation to be structured in this way for the following additional reasons:

to facilitate its comparison with the structure of the other Thermodynamic Principles (especially the First and the Second ones);

to include the most general possible conditions pertaining to very Complex Systems, concerning both their time evolution behavior and space structure organization.

The latter consideration especially refers to the fact that, if we want to study a living organism (e.g. a plant or an animal) or even an organ (e.g., the liver or the brain) as a whole, the presence of billions and billions of cells suggests we consider a *continuous* system rather than a discrete one made up of n parts. This implies that Eq. (2.3) becomes more general if re-written in terms of integrals instead of summations. However, even if formulated in such a way, it can always be easily reduced into discrete terms, if necessary. And even if there might be some discontinuity conditions, they can always be dealt with in the frame of Lebesques' theory of integrals (Kolmogorov and Fomin, 1980), which undoubtedly covers all the cases usually considered in Literature on the subject.

Under such assumptions, Eq. (2.3) can be re-written in the following synthetic and compact form

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3 V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3 V \tag{4.1}$$

where

φ_v^* = the "*equivalent*" Source Term per unit volume (see Eq.(1.4) in discrete terms);

Γ = the *local* structural amplification and re-normalization factor (corresponding to the product of the coefficients γ_k^* and γ_k in the case of discrete form (see Eq. (2.3)), which also accounts for the structural variations with time;

$$em_v^* = em_{v,m} + em_{v,q}^+ + em_{v,w}^+ \tag{4.2}$$

in which

$em_{v,m} = C\,?\rho\,?ex$ is the Emergy per unit volume associated to the mass (thus transportable by mass flows)3

$em_{v,q}^+$ = the Emergy per unit volume associated to heat source terms

$em_{v,w}^+$ = the Emergy per unit volume associated to work source terms4.

Under such conditions the Maximum Em-Power Principle may be mathematically formulated as follows

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \to Max \quad , \quad \forall D^*(t) \mid \; S_U(t) \tag{4.3}$$

that is valid for any Domain (D^*) belonging to Universal Space ($S_U(t)$). Such a principle, in the light of the previous considerations, may be verbally formulated as follows: "Every System tends to organize its internal structure to generate progressively increasing spring-Emergy levels in order to maximize the flow of processed (or "useful") Emergy".

5. GENERAL CONSIDERATIONS ON THE MATHEMATICAL FORMULATION OF THE MAXIMUM EM-POWER PRINCIPLE

Formulation (4.3) may be seen as constituted by three parts:

1st part:

$$\frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \to Max \quad , \qquad \forall D^*(t) \subseteq S_U(t) \tag{5.1}$$

which corresponds to the usual definition of the Principle in terms of phenomenological *effects*:

"Every System tends to maximize the flow of processed (or "useful") Emergy";

2nd part:

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V \to Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \tag{5.2}$$

which points out the internal *causes* of such effects: "Every System tends to organize its internal structure for progressively increasing spring-Emergy levels";

3rd part:

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V \overset{\rightarrow}{=} \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \to Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \tag{5.3}$$

which emphasizes that its mathematical structure is a logical consequence of the first two mentioned parts and points out the existing direct relationship between internal *causes* (5.2) and phenomenological *effects* (5.1). The symbol $\overset{\rightarrow}{=}$ has been specifically introduced to emphasize the versus of the equivalence which goes *from causes to effects*.5 Thus an alternative verbal formulation which is able to include *both* the first and the second side of Eq. (5.3) could be the following one: "Every System tends to maximize its internal structure level in order to maximize its Spring-Emergy Flow which, in turn, maximizes the flow of processed Emergy". In addition it is also worth pointing out that Eq. (4.3) (or Eq. (5.3)) expresses a *tendency* Principle. This implies that:

i) the symbol Max has to be considered, in general, as an *Extremum*;

ii) thus in the long run, Eq. (4.3) can also be written as follows

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \geq 0 \quad , \qquad \forall D^*(t) \subseteq S_U(t) \tag{5.4}$$

where the sign equal holds only when the maximum is actually achieved;

iii) for very complex systems the dynamic behaviour is usually controlled by very high time constants, so that the slope of the increasing trend could be so slow that, in a given time-space window of analysis, it would be possible to assume that

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3 V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3 V \cong 0 \qquad (5.5).$$

It is also worth mentioning that the previous expression (4.3) constitutes the formulation of the M. Em-P. P. in its *general version* that could also be said *in a weak sense*. In fact it is also possible to consider another and more cogent definition (*in a strong sense*) as follows

$$\frac{d}{dt} \int_{D^*(t)} \Gamma \varphi_v^* d_3 V = \frac{d^2}{dt^2} \int_{D^*(t)} em_v^* d_3 V \geq 0 \quad , \quad \forall D^*(t) \subseteq S^* \subset S_U(t) \qquad (5.6)$$

which is valid however only for particular subsets (S^*) of Universal Space $S_U(t)$.

On the basis of the previous results we can now analyze the relationship between the M. Em-P. Principle and the other Thermodynamic Principles.

6. THE SECOND PRINCIPLE IN THE LIGHT OF THE M. EM-P. PRINCIPLE

As an introductory aspect let us consider a simple example: a Complex System, made up of n discrete sub-systems, in steady state conditions. In this case Eq. (2.3) may be re-written as follows

$$(1+\varphi^*) \cdot \sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}m(y_l) \qquad (6.1)$$

In fact, on the basis of Eqs. (1.3) and (1.4), the comprehensive "*equivalent*" Source Term (Φ^*) can be expressed in terms of the equivalent input Emergy flow, through an equivalent specific amplification (or generative) efficiency φ^*:

$$\Phi^* = \sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) = \varphi^* \cdot \sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) \qquad (6.2)$$

Such a procedure, which is *always* possible on the basis of the n Emergy Balance equations describing the n considered sub-systems (see Giannantoni, 2001d), becomes particularly simple in steady state conditions (Giannantoni, 2000a).

If we now assume, for the sake of simplicity, that the System has just one input, we get

$$(1+\varphi^*) \cdot \dot{E}m(u) = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}m(y_l) \qquad (6.3)$$

If (for ulterior simplicity) we assume equal co-production coefficients (β_l), equal renormalization factors (β_l^*) and their product (β_l^* ?β_l) equal to 1, we have

$$Tr(y_l) = Tr(u) \cdot \frac{Ex(u)}{\sum_{l=1}^{p} Ex(y_l)} \cdot (1+\varphi^*) \qquad (6.4).$$

Eq. (6.4) has a fundamental importance because it shows that the Transformity of any output quantity is an amplification of the input Transformity as a consequence of*two* distinct reasons:

i)the dissipation of Exergy due to the Second Thermodynamic Principle ($\sum_{l=1}^{p} Ex(y_l) < Ex(u)$);

ii) the amplification factor $(1+\varphi^*)$, which indicates the global gain specifically due to the generation of Emergy on behalf of internal Source Terms.

But what is really now worth pointing out is that, for very Complex Systems characterized by a lot of internal co-generation and/or interaction processes, the contribution given by the amplification factor $(1+\varphi^*)$ is generally *much higher* than the one due to Exergy losses. In addition, what is even more important is that the amplification due to *Emergy* Source Terms is *in-dependent* from the other one: the former can also be present both in ideal Systems (that is even if Exergy losses had been absent) and in real Systems (even if characterized by an increase of Exergy).

This result can be considered as a first indication of the fact that the M. Em-P. P. is *independent* from the Second Principle and, what is even more important, it indicates the *effective reason* for the increasing order in self-organizing Systems, whereas the (habitually) associated loss of Exergy constitutes only a *concomitant circumstance*.

Such a conclusion can be easily (and more rigorously) drawn by starting from a very general point of view, that is by starting from the general formulation (4.3) of the M. Em-P. Principle (as we will see in par. 9). Let us now consider another important aspect: how it is possible to obtain the traditional Exergy Balance Equation from the one that expresses the M. Em-P. Principle. If, in fact, we consider Systems described only in terms of a traditional Thermodynamic approach (that is by neglecting any Emergy Source term, although ever present), our description reduces either to the classical Exergy Balance Equation or the Energy Balance Equation, according to the specific *distinct assumptions* considered in the two different mentioned cases.

The Exergy Balance Equation may be obtained by remembering that the irreversibility flow terms (mainly due to internal, but also to external losses) which can be expressed as

$$\frac{d}{dt}\int_{D^*(t)} em^*_{v,irr}\, d_3V = \frac{\partial}{\partial t}\int_{D^*(t)} C\cdot\rho\cdot ex^*_{irr}\, d_3V + \int_{\partial D^*(t)} C\cdot\rho\cdot ex^*_{irr}\cdot v_{rn}\, d_2S \tag{6.5}$$

are generally neglected in Emergy Analysis on the basis of the assumption that their specific co-production coefficients are equal to zero (see Giannantoni, 2000a). If, vice versa, we continue to consider such contributions, but we distinguish between their *conceptual existence* and the fact that their pertinent local co-production coefficients are everywhere *numerically equal to zero* $(c_{irr}(x,y,z,\tau)=0)$ (ib.), we may recognize that the Exergy Balance Equation can be thus easily obtained from the Maximum Em-Power Principle by assuming that all the co-injection and co-production factors are equal to 1. Such an assumption corresponds to considering *independent inputs* and all the *outputs as splits*. Under these conditions, and without any Emergy Source Term $(\Phi^*=0)$, Eq. (4.3) becomes, in explicit terms

$$0 = \frac{\partial}{\partial t}\int_{D^*(t)} \rho\cdot(h_{kz}-Ts)\cdot d_3V + \int_{\partial D^*(t)} \rho\cdot(h_{kz}-Ts)\cdot v_{rn}\, d_2S + \int_{D^*(t)} \dot{q}^+_v\,\theta\cdot d_3V + \int_{\partial D^*(t)} \dot{q}^+_s\,\theta\, d_2S +$$
$$+ \int_{D^*(t)} \dot{w}^+_{v,p}\, d_3V + \int_{\partial D^*(t)} \dot{w}^+_s\, d_2S + \frac{\partial}{\partial t}\int_{D^*(t)} \rho\cdot ex^*_{irr}\, d_3V + \int_{\partial D^*(t)} \rho\cdot ex^*_{irr}\cdot v_{rn}\, d_2S \tag{6.6}$$

which exactly expresses the Exergy Balance Equation, usually written as follows

$$\int_{D^*(t)} \dot{q}_v^+ \theta d_3V + \int_{\partial D^*(t)} \dot{q}_s^+ \theta d_2S + \frac{\partial}{\partial t} \int_{D^*(t)} \rho \cdot (u_{kz} - Ts) \cdot d_3V + \int_{\partial D^*(t)} \rho \cdot (h_{kz} - Ts) \cdot v_{rn} d_2S =$$

$$+ \int_{D^*(t)} \dot{w}_v^+ d_3V + \int_{\partial D^*(t)} \dot{w}_s^+ d_2S - \frac{\partial}{\partial t} \int_{D^*(t)} \rho \cdot ex_{irr}^* \cdot d_3V - \int_{\partial D^*(t)} \rho \cdot ex_{irr}^* \cdot v_{rn} d_2S \qquad (6.7)^6$$

where

θ is the generalized Carnot coefficient.

7. THE FIRST PRINCIPLE IN THE LIGHT OF THE M. EM-P. PRINCIPLE

As far as the relationship between the First Principle and the M. Em-P. P. is concerned, we may easily obtain the mathematical formulation of the former from that of the latter by following a methodological procedure analogous to the one followed in the case of the Second Principle. The only difference now is that the First Principle does not take into consideration what happens *inside* the System (which is in fact considered as a "black box"), but only accounts for quantities measured on the System frontier as simple *additive* contributions. Under such conditions, the local co-injection and co-production factors $c_{irr}(x, y, z, \tau)$ due to the various irreversibilities, are assumed to be equal to zero in the whole volume not because it is assumed that they do not contribute to the Balance Equation (like in the case of Emergy Balance), but because they are not "seen" in the assumed balance perspective. If then, in addition, we do not consider any form of Emergy (although ever present), that is $\Phi^* = 0$, we can easily get, in explicit terms

$$0 = \frac{\partial}{\partial t} \int_{D^*(t)} \rho \cdot h_{kz} \cdot d_3V + \int_{\partial D^*(t)} \rho \cdot h_{kz} \cdot v_{rn} d_2S + \int_{D^*(t)} \dot{q}_v^+ d_3V + \int_{\partial D^*(t)} \dot{q}_s^+ d_2S +$$

$$\int_{D^*(t)} \dot{w}_{v,p}^+ d_3V + \int_{\partial D^*(t)} \dot{w}_s^+ d_2S \qquad (7.1)$$

which is exactly the Energy Balance Equation, usually written as follows

$$\frac{\partial}{\partial t} \int_{D^*(t)} \rho \cdot en \cdot d_3V + \int_{\partial D^*(t)} \rho \cdot en \cdot v_{rn} d_2S = \int_{D^*(t)} \dot{q}_v^+ d_3V + \int_{\partial D^*(t)} \dot{q}_s^+ d_2S +$$

$$\int_{D^*(t)} \dot{w}_{v,nc}^+ d_3V + \int_{\partial D^*(t)} \dot{w}_{s,nc}^+ d_2S \qquad (7.2)^7.$$

It is now really important to point out that the methodological procedure previously followed in order to obtain both the Exergy Balance Equation and (analogously) the Energy Balance one from the mathematical formulation of the Maximum Em-Power Principle cannot be thought of as a *deduction* of the former equations from the latter, but rather as a "*reduction*" of the latter to the former. The considered Principles in fact *always remain in-dependent* from each other. The procedure followed only shows in what *reductive* perspective (and associated *limiting* assumptions) the M. Em-P. P. can be thought of as being "equivalent" to the two basic well-known traditional Thermodynamic Principles.

Now, before analyzing the Third Thermodynamic Principle in the light of the M. Em-P. Principle, it is worth dealing with the Minimum Action Principle, because it is closely related to the First Principle.

8. THE *MINIMUM* ACTION PRINCIPLE IN THE LIGHT OF THE *MAXIMUM* EM-POWER PRINCIPLE

The Minimum Action Principle[8] is a well-known in Classical Mechanics. It deals with Systems characterized by kinetic and potential Energy, in the absence of any dissipative process. Under such conditions it states that, for each System, there exists a function $L(q,\dot{q},t)$ defined as

$$L(q,\dot{q},t) = T - U \tag{8.1}$$

which satisfies the following condition

$$\delta \int_{t_1}^{t_2} L(q,\dot{q},t)\cdot dt = \delta \int_{t_1}^{t_2} (T-U)\cdot dt = 0 \tag{8.2}$$

where

q = set of geometrical coordinates

$\dot{q}$ = set of their time derivatives

T = kinetic Energy

U = potential Energy

and the symbol δ represents the *variation* of the *first order* (often simply named as *variation*) of the integral when the function $q(t)$ is replaced by $q(t)+\delta q(t)$, where $\delta q(t)$ is a function which is sufficiently small in the whole time interval $[t_1,t_2]$. The Minimum Action Principle is very useful to describe the time-space evolution of a Complex System made up of n parts. In this case, and in the (usual) hypothesis of time *uniformity*, it is perfectly equivalent to the Energy Conservation Principle. In fact, in these conditions, the Lagrangian function $L(q,\dot{q},t)$ does not depend explicitly on the time and Eq. (8.2) can be re-written as follows (Landau and Lifchitz, 1969)

$$\delta \int_{t_1}^{t_2} L(q,\dot{q},t)\cdot dt = \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} \right) \cdot \partial q \cdot dt = 0 \tag{8.3}.$$

The equations of motion

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0 \tag{8.4}$$

derived from Eq. (8.3) imply, as a main consequence, that one particular integral of the first order (called Energy) is constant

$$En = T + U = const \tag{8.5}.$$

In other words the Minimum Action Principle describes the time-space evolution of a System (see Eqs. (8.4)) which involves a continuous transformation between kinetic and potential Energy contributions (see Eq. (8.2)), but in such a way as to satisfy the condition that their *sum* is at any time kept constant (Eq. (8.5)). Consequently the Maximum Em-Power Principle is by no means in contrast with such a principle: in fact it not only *confirms* the Minimum Action Principle, but also *adds* something more. As far as the former aspect is concerned, what we said in par. 7 may be considered as being already sufficient. As far as the latter is concerned, we should remember that the Minimum Action Principle describes the System only in terms of *additive* contributions of mechanical Energy, whereas the Maximum Em-Power Principle is able to describe phenomena in which *the whole is more than its parts*. As an example we can consider two elements that generate a new entity described by a *binary function*: in this case the Emergy output, obtained as a solution of a differential equation of *fractional* order (see Giannantoni, 2001d), is higher than the input Emergy of its corresponding components. The case of a photon (considered as made up of a positron and an electron (Giannantoni 2000b, 2001a,b) and the examples analyzed in Giannantoni (2001d) can be significantly explanatory. From a general point of view however, if we consider an *isolated* System $\Sigma^{\bullet}$, we can write

$$\int_{\Sigma^*(t)} \Gamma \varphi_v^* d_3 V = \frac{d}{dt} Em_\Sigma \rightarrow Max \tag{8.6}.$$

Consequently, by remembering (see Ref. [10]) that we can always write the System Emergy as

$$Em_\Sigma = Tr_\Sigma \cdot Ex_\Sigma \tag{8.7},$$

in the long run we have

$$\frac{d}{dt} Em_\Sigma = \frac{d}{dt} Tr_\Sigma \cdot Ex_\Sigma + Tr_\Sigma \cdot \frac{d}{dt} Ex_\Sigma \geq 0 \tag{8.8}$$

Consequently, there are two different possibilities: a) if we analyze the System in terms of only Energy additive contributions (like in the case of the Minimum Action Principle), there are no Emergy Source Terms. At the same time this implies that the System Transformity Tr_Σ is constant and Eq. (8.8) is valid with the sign equal, that is

$$\frac{d}{dt} Ex_\Sigma = 0 \tag{8.9}$$

which evidently corresponds to the Energy Conservation Principle for non-dissipative systems;

b) but we will immediately see (in the next paragraph) that Emergy may generally be increasing even if Eq. (8.9) holds. This means that the Maximum Em-Power Principle, as a Quality Principle, adherently *contemplates the co-existence* of the Minimum Action Principle (see Eq. (8.8)), which can consequently still be considered as being a *quantitatively independent* Principle.

9. *ORDER* AND *DISORDER* IN THE LIGHT OF THE M. EM-P. PRINCIPLE

The M. Em-P. Principle also throws *new light* on the relationship between *order* and *disorder* in the Universe. To this purpose it is useful to start from the application of Eq. (4.3) to the Whole Universe, thought of as an *isolated* System (such an application will also clearly illustrate the result synthetically mentioned in par. 6). Under such conditions we have (see also Eq. (8.6))

$$\int_{S_U(t)} \Gamma \varphi_v^* d_3 V = \frac{d}{dt} Em_U \rightarrow Max \tag{9.1}.$$

Consequently, by remembering (see Giannantoni 2000a) that we can always write the Universe Emergy as

$$Em_U = Tr_U \cdot Ex_U \tag{9.2}$$

and by taking the total derivative, we can then obtain (in analogy to E. (8.8))

$$\frac{d}{dt} Em_U = \frac{d}{dt} Tr_U \cdot Ex_U + Tr_U \cdot \frac{d}{dt} Ex_U \tag{9.3}.$$

If we now consider that in the long run we may use Eq. (5.4), we can write

$$\frac{1}{Tr_U} \cdot \frac{d}{dt} Tr_U \geq -\frac{1}{Ex_U} \cdot \frac{d}{dt} Ex_U \tag{9.4}$$

which, on the basis of Eq. (6.4), is always valid in the following form

$$\frac{1}{Tr_U} \cdot \frac{d}{dt} Tr_U >> -\frac{1}{Ex_U} \cdot \frac{d}{dt} Ex_U \tag{9.5}.$$

If then, on the basis of the Second Principle, we assert that

$$\lim_{t \to +\infty} \Delta S = \lim_{t \to +\infty} - \int_{t_0}^{t} \dot{Ex}_U(\tau) d\tau \to Max \tag{9.6}$$

this implies that

$$(-\frac{d}{dt} Ex_U) = 0 \lim_{t \to +\infty} \tag{9.7}.$$

Consequently Eq. (9.5) enables us to assert that Tr_U is always increasing (according to a logarithmic trend) even in these *extreme conditions*, because it is always

$$\frac{1}{Tr_U} \cdot \frac{d}{dt} Tr_U >> 0 \lim_{t \blacklozenge +\times} \tag{9.8}.$$

This result continues to be true in the light of the *Third Thermodynamic Principle* too.
In fact, if we formulate the Third Principle in terms of Exergy variations, that is

$$\lim_{T_0 \to 0} \Delta S = -\lim_{T_0 \to 0} \frac{\Delta Ex}{T_0} = 0 \tag{9.9}$$

(where T_0 is the absolute Temperature of the Universe), we are able to assert that, even if the Exergy variations are infinitesimal of higher order with respect to T_0, this fact does non imply any superior limit to the increasing Universe Transformity Tr_U. So that the M. Em-P. Principle allows us to conclude that: i) not only does the *meta*-mechanical *order* (represented by Tr_U) increase much more rapidly than the mechanical *disorder* (expressed by the increase of Entropy); ii) but even when the latter is near its maximum, there is always a possible increase of Tr_U (even in the presence of an extremely limited availability of residual Exergy), because most of the increase in Tr_U (as already shown) is *independent* from Exergy variations, which always represent only a *concomitant circumstance*.

10. THE SO-CALLED FIFTH PRINCIPLE IN THE LIGHT OF THE MAXIMUM EM-POWER PRINCIPLE

The results achieved in the previous paragraph allow us to show that the *hierarchical order of the Universe* (the so-called Fifth Principle) can also be "obtained" on the basis of the M. Em-P. Principle, but such process of "deduction" is substantially different from a traditional deductive process. Such a Principle in fact cannot be thought of as being a *mathematical corollary* to the Maximum Em-Power Principle, but as a sort of *crowning* of the former, because it presents a *higher* Quality content in its conclusive assertions.

The Fifth Principle in fact asserts that "Energy flows of the Universe are organized in Energy transformation hierarchy. Position in the Energy hierarchy is measured with Transformities" (Odum, 1994b). Alternatively, it can also be enunciated as follows: "The Universe is hierarchically organized and a manifestation of Energy. Transformity is a measure of the hierarchy of Energy" (ib.).

The latter of the two quoted equivalent versions is especially indicated for our considerations. In fact, if we consider the Universe as being a Whole, which is (only ideally) repeatedly sub-divided into *two* different parts, and in such a perspective we first apply Eq. (9.1) to the *whole* System

$$\int_{D_1^*(t)} \Gamma\varphi_v^* d_3V + \int_{D_2^*(t)} \Gamma\varphi_v^* d_3V = \frac{d}{dt}\left(Em_1 + Em_2\right) \to Max\,, \quad D_1^*(t) \cup D_1^*(t) \equiv S_U(t) \qquad 10.1)$$

and contemporarily we apply the same equation to the
two distinct parts (e.g. in a discrete form, such as Eq. (2.3)), it is easy to show that: i) while the global Transformity of the Universe (Tr_U) is generally increasing (see also par. 9), ii) the rates of the Accumulated Emergy pertaining to the two sub-systems are not identical: in fact they depend on different Emergy Source distributions and generally different Emergy interchange flows. If we then consider the corresponding equations that express such sub-system variations in terms of Transformity and Exergy (in analogy to Eq. (9.3) which is valid for the whole System), we can easily conclude that, even if (by hypothesis) the sub-system Transformities are originally equal, in general they progressively tend to differ with time. This evidently shows that there is a hierarchy between the Transformities pertaining to the two sub-systems each time considered. Consequently there is a distribution of Transformities in the Universe, which describes the content of information (or the degree of organization) of all the various possible subsystems considered. Such a distribution is fundamentally due to *two* distinct reasons: the distribution of Emergy Sources (main cause), which is also responsible for Emergy interchange flows, and the distribution of Exergy variations (as concomitant effects) associated to Energy transformations. So that, on the basis of considerations similar to those that have led us to Eq. (6.4), we can also assert that the effects due to Emergy Sources are those which are generally *dominant* and, above all, *independent* from those which are due only to the above-mentioned associated circumstances. Consequently, the *hierarchical organization* logically derived from Exergy variations (generally proportional to a *time-space scale*) is thus only a basic (or a *first order*) distribution (as a consequence of the Second Principle). In fact this basic effect is contemporarily *modulated* by a superimposed hierarchical distribution due to Spring-Emergy Sources (Fourth Principle), which is generally much more influent than the former. This allows us to reach the following general conclusions: i) the hierarchical organization *tendency* of the Universe is generally *increasing*; ii) it is *essentially* based on the Forth Principle, but its statement shows an *increased Quality content* so that such a "de-ductive" process could be more appropriately named (by means of a neologism) as an "*over-deduction*" ; iii) the Fifth Principle is thus a *substantial step ahead with* respect to the *traditional hierarchy* of the various well-known Thermodynamic Principles. It Represents *a new jump of Quality* in such a hierarchy and, consequently, an explicit harmoniously adherent *crowning* of the M. Em-P. Principle.

In the light of the previous considerations we could thus reformulate such a Principle as follows: "The Universe is hierarchically organized. Its organization order level has a generally *increasing tendency* and is a manifestation of both Emergy Source distribution and Energy transformations. Transformity is a *comprehensive* measure of such a *dynamic* (or, rather, *thermodynamic*?) hierarchy".

At this stage, as a consequence of all the previous paragraphs, we may ask the following (and, in a certain sense, already anticipated) basic question:

11. IS THE MAXIMUM EM-POWER PRINCIPLE A *THERMODYNAMIC* PRINCIPLE?

In order to answer this question in a complete and correct way, we have to deal with another really preliminary question: as to whether Emergy is a *state variable* or not.

To this purpose we may re-structure Eq. (4.1) in an appropriate way or, to simplify further, we may re-structure Eq. (2.3) in the following form;

$$\frac{d}{dt}\int_{D^*(t)} C\rho ex \cdot d_3V = \sum_{k=1}^{n} \gamma_k^* \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) + \sum_{j=1}^{m'} \alpha_j^* \alpha_j \cdot \dot{E}m\left(u_{j,q,w}\right) - \sum_{l=1}^{p'} \beta_l^* \dot{\beta}_l \cdot Em\left(y_{l,q,w}\right) \qquad (11.1),$$

where the last two terms refer to input and output Emergy contributions associated to heat and work flows (in fact mass flow contributions are included in the term on the left side of the equation).

Equation (11.1), if thought of as belonging to the complete set of balance equations (mass, momentum, Energy, Exergy, etc.)describing the behavior of the System, contributes (directly or in its integrated form) to define State $\underline{x}(t)$ of the System, which satisfies the required fundamental conditions of *uniqueness*, *causality*, *consistency* and *separability* (see Appendix 1). This implies that Emergy, defined by Eqs. (1.1) and (1.2), satisfies (through Eq. (11.1)) *all the properties* required in order to be a *state variable*.

Furthermore we have also shown that, under specific assumptions, the well-known Thermodynamic Principles can be obtained from the M. Em-P. Principle. Such a process, however, as previously stated, should not be considered as a *deduction* of the former from the latter, but rather as a sort of "*reduction*" of the latter to the former. In other words such a procedure gives us results which are strictly equivalent to the classical formulations only in *quantitative* terms, but not in Quality terms: in fact there is always the presence of dimensional factors characteristic of Emergetic Algebra that retain their specific meaning even if they are assumed to be equal to 1. This means that we are able to obtain the above-mentioned Classical Principles (in their *quantitative version*) by accounting for only that fraction of Emergy which corresponds merely to the *mechanical relationship* between the different parts of the System: this can be either the part which can be integrally transformed into mechanical work (in the case of the Second Principle) or the part which pertains to every Energy relationship, in its various forms, independently on the reciprocal and complete transformability of one form into another (in the case of the First Principle). In other words the M. Em-P. Principle *never loses* its capability of accounting for that *meta-mechanical* relationship (that is: *beyond* the mere mechanical aspect) which always characterizes physical phenomena, the presence of which is always recalled by the associated dimensional factors, though (or even if) we forget or neglect its presence when accounting for only mechanical entities.

This fact should already be sufficient to assert that the M. Em-P. P. is *not* a Thermodynamic Principle (in the usual sense of the term), because in actual fact it is *much more*. In fact it cannot be, strictly speaking, referred to as *Thermodynamic* because it does not deal only with the mere transformability of heat into mechanical work or transformability of mechanically equivalent Energy from one form into another. On the other hand it can *still* be termed as "Thermodynamic" (in a more general sense) if we recognize that there are forms of "work" that go *beyond* the mechanical aspects (this is why they have been previously named *meta*-mechanical), found especially in living systems,

human systems, information, artificial intelligence and so on. There are in fact many forms of "Energy" (understood in its widest meaning) capable of sustaining and driving physical phenomena. All of them can be object of analysis by means of the Maximum Em-Power Principle: the only necessary requisite is their observability, describability and measurability.

12. CONCLUSIONS

By starting from a rigorous mathematical definition of Emergy (Eqs. (1.1), (1.2)) and a General Emergy Balance Equation appropriately restructured (Eq. (2.3)), we have shown that: i) A mathematical formulation of the Maximum Em-Power Principle can be stated in the form given by Eq. (4.3); ii) As a first result, such a formulation contributes to a clear and rigorous definition of the meaning of "useful" (or "processed") Emergy; iii) The real novelty, however, of such a formulation, apart from the mathematical aspect, is its constitution in *three fundamental parts*: *causes*, *effects*, and their relationship understood as being a

common tendency; iv) As a

tendency Principle, the M. Em-P. P. can also be formulated both in a *weak sense* (as a general principle) and in a *strong sense* (in particular cases); v) If understood as a *general principle*, the given mathematical formulation allows us to enlighten the previous and well-known Thermodynamic Principles, without ever losing its Quality properties, even if it is applied according to a "reduction" procedure. In fact we have shown, in particular, that: vi) The First and the Second Principles can be obtained (independently from each other) from the given mathematical formulation of the Maximum Em-Power Principle; vii) Such a "derivation" however cannot be thought of as being a *deduction* of the former Principles from the latter, but rather as a "*reduction*" of the latter to the former; viii) The three considered Principles in fact *always remain in-dependent* from each other. The procedure followed only shows in what *reductive* perspective (and associated *limiting* assumptions) the M. Em-P. P. can be thought of as being "equivalent" to the two basic well-known traditional Thermodynamic Principles; ix) The *Maximum* Em-Power Principle, as a Quality Principle, adherently *contemplates the co-existence* of the Minimum Action Principle, so that this can still be considered as being an *independent* (though *only quantitative*) Principle; x) The mathematical formulation of the M. Em-P. Principle also allows us to see the relationship between *order* and *disorder* in a new light: in fact, the *meta*-mechanical *order* (represented by $\mathcal{T}r$) increases much more rapidly than the mechanical *disorder* (expressed by the increase in Entropy); in addition, most of the increase in $\mathcal{T}r$ is *independent* from Exergy variations, which always represent only a *concomitant circumstance*; xi) This clearly shows how the M. Em-P. Principle, even if it accounts for Exergy dissipations (which are ever present in any process), is *conceptually independent* from the Second Principle (which only deals with such concomitant circumstances); xii) The Fifth Principle, vice versa, although "*obtainable*" from the mathematical formulation of the Maximum Em-P. Principle, cannot be considered as a mathematical *corollary* to the latter, but as an exceeding Quality *crowning* of the Maximum Em-P. Principle; xiii) The *order* in the Universe (asserted by such a Principle), hierarchically associated to different time-space scales and basic Exergy variations, is in fact mainly characterized by a superimposed ordering distribution fundamentally due to the Spring-Emergy Sources, when these are seen as part of a Whole; xiv) We also demonstrated that Emergy has all the properties of a *state variable*; xiv) This paved the way to a clear assertion in favor of the M. Em-P. P. as a *Thermodynamic Principle*: both when it is understood in the "reductive" traditional sense and in the *new* much more *general Thermodynamic perspective* opened by the concept of *Emergy*; xvi) As far as the last aspect is concerned, we also emphasized the widest perspective of such a Principle in dealing with *observable*, *describable* and *measurable* phenomena. Consequently, in this context, it has an extremely wide generality and an almost unlimited practical application; xvii) Finally, the mathematical formulation of M. Em-P. Principle enabled us to show that such a Principle, when seen in the light of the hierarchical progressive increase in Quality content, in the successive passages from the traditional Thermodynamic Principle to the Fifth Principle, represents a *new* fecund *starting point* rather than an important *point of arrival*, especially

because of the above-mentioned properties which have been clearly pointed out as consequence of its mathematical formulation.

To conclude this paper it is worth pointing out that such a mathematical formulation of the Maximum Em-Power Principle is not only the result of a persistent effort constantly gazing forth to give an *elegant* form to this *Quality* Principle: in other words, *a quality language for a quality principle*. It is also, and at the same time, a sort of *hymn to Quality*.

Quality, in fact, which always plays such a fundamental role in Emergy Analysis, cannot be, strictly speaking, "derived". It can only be recognized, described, accepted. It cannot be derived in any case whatsoever, because it is fundamentally, by itself and in itself, in-derivable. Quality in fact is always *emerging*, *generative*, *primary*. It simply *appears*: shows itself, presents itself, reveals itself, and it is always source of astonishment, fascination, charm. It certainly has a foundation: this is given by the *quality of the presuppositions* from which it originates; but the "process of emerging spring genesis" is the one which always remains, specifically, *in-derivable*.9

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APPENDIX 1. EMERGY AS A STATE VARIABLE

The problem can be seen in the most general context of Dynamic Systems Theory. In such a context, in fact, the approach is simplified because Emergy Analysis is specifically oriented at describing what happens

inside the System (it is not a "black box" analysis), so that we can immediately start from a System description which corresponds to the *input-state-output* approach. Thus we first consider the set of differential equations that describe the Complex System under consideration, that is: continuity, momentum, Energy, Exergy, etc., and Emergy Balance Equations written for each sub-System. Under such conditions it is very easy to identify those variables that are potentially able to play the role of *state variables*. One of those could be Total Emergy (obviously related, as we already know, to the specific Emergy of each sub-system).

If we then consider the classical *input-state-output* mathematical representation of the System in the pertinent form given through the *state transition function*

where $$\psi : (TxT)^{*} x \underline{X} x \underline{U} \rightarrow \underline{X}, \tag{A.1}$$

and the *output transformation function*

$$\underline{y}(t) = \eta(t, \underline{x}(t), \underline{u}(t)) \qquad \text{where} \qquad \eta : Tx \underline{X} x \underline{U} \rightarrow \underline{Y} \tag{A.2}$$

where $\underline{u}(t)$, $\underline{x}(t)$, $\underline{y}(t)$ are the *input*, *state* and *output* vectors respectively such as $\underline{u}(t) \subseteq \underline{U}$, $\underline{x}(t) \subseteq \underline{X}$, $\underline{y}(t) \subseteq \underline{Y}$ (while $\underline{U}, \underline{X}, \underline{Y}$ are the input, state and output spaces respectively), we can easily verify that Eq. (11.1) satisfies the above-mentioned conditions such as (see Ruberti and Isidori, 1969):

a) *uniqueness* and *causality*

$$\psi(t, t_0, x_0, \underline{u}_1 \underset{[t_0, t)}{}) = \psi(t, t_0, x_0, \underline{u}_2 \underset{[t_0, t)}{}) \tag{A.3}$$

$$\forall (t,t_0) \in (TxT)^*, \quad \underline{x}_0 \in \underline{X}, \quad \forall \underline{u}_1, \underline{u}_2 \in \underline{U}: \ \underline{u}_1{}_{-[t_0,t)} = \underline{u}_2{}_{-[t_0,t)}$$

that is: identical inputs lead the System to the same and unique final state value;

b) *consistency*

$$\underline{\psi}(t_0, t_0, \underline{x}_0, \underline{u}) = \underline{x}_0 \tag{A.4}$$

$$\forall t_0 \in T, \quad \forall \ \underline{x}_0 \in \underline{X}, \quad \forall \underline{u} \in \underline{U}$$

that is: the transition into the initial state coincides with the initial state value;

c) *separability*

$$\underline{\psi}(t, t_0, \underline{x}_0, \underline{u}_{-[t_0,t)}) = \underline{\psi}(t, t_1, \underline{\psi}(t_1, t_0, \underline{x}_0, \underline{u}_{-[t_0,t_1)}), \underline{u}_{-[t_1,t)}) \tag{A.5}$$

$$\forall (t,t_0) \in (TxT)^*, \quad \forall t_1 \in (t_0,t), \quad \forall \underline{x}_0 \in \underline{X}, \quad \forall \underline{u}_{-[t_0,t)} \in \underline{U}$$

that is: the state evolution can be analyzed by means of a finite sequence of successive steps.

It is also worth adding that the property of separability has already been considered as a fundamental assumption in order to define a reference level of Emergy (e.g., Solar Emergy). In fact such a property immediately allows us to write

$$Em^*(t) = Em_0 + \int_{t_0}^{t} \dot{Ex}_{eq}(\tau) d\tau \tag{A.6}$$

(see also Giannantoni, 2000a, Eq. (2.3)).

(Footnotes)

1 Such a concept, already introduced in Giannantoni (2000a), will be analyzed in detail in the companion paper (Giannantoni, 2001d) presented in this Conference, especially as far as its profound meaning and wide consequences are concerned.

2 In this form each input and internal contribution to the System has its corresponding effect*directly* and *exclusively* expressed in terms of output quantities. Under these conditions we always have $\left(\beta_l^* \cdot \beta_l\right) \geq 1$, for l = 1,2,Ö p.

3 For the sake of generality, in open systems the specific mass Exergy is defined as

$ex = (h - T_0 s) + \frac{1}{2} v^2 + gz$, that is it includes the kinetic and potential terms.

4 The apex "+" indicates a positive quantity when *furnished* by the System, while its algebraic sign specifies the exact versus. Obviously we may use, if needed, all the consistent transformations such as $-A^+ = +A^-$ and so on. Such a convention is particularly useful for a successive comparison with the Second Principle formulated in terms of Exergy Balance Equation (see par. 6). Generally speaking we could say that the convention suggests the *perspective* according to which one should evaluate the actual versus of each considered flow.

Moreover, the term pertaining to work is understood as including a specific contribution due to pressure work such as $\int_{D^*(t)} \frac{\partial p}{\partial t} d_3V$ in order to have the same expression for specific mass Exergy (ex) both in the volume and surface integrals and in the case of both a Eulerian and Lagrangian description. At the same time the System is assumed to be subjected to conservative force fields which are assumed to be constant in time (as usually happens).

5 Even if such a symbol is not adopted in writing the following equations, it will always be understood as being substituted by the *convention* according to which the equations will be (generally) written: the *left* side will represent the *causes* and the *right* side will represent the corresponding *effects*. Such a physical-mathematical convention (*from left to right*) is in some way analogous to the *topological* convention adopted in Emergy Analysis *System Diagrams.*

6 The difference between the presence of $h_{kz} = h + \frac{1}{2}v^2 + gz$ in Eq. (6.6) and of $u_{kz} = u + \frac{1}{2}v^2 + gz$ in Eq. (6.7) depends on the fact that the term $\overset{?}{w}{}^{+}_{v,p}$ includes the pressure work mentioned in note 4, whereas the term $\overset{?}{w}{}^{+}_{v}$ does not.

7 The difference between Eq. (7.1) (in which there is $h_{kz} = h + \frac{1}{2}v^2 + gz$) and Eq. (7.2) (in which there is $en = u_{kz} = u + \frac{1}{2}v^2 + gz$) depends on the fact that the terms $\overset{?}{w}{}^{+}_{v,nc}$ and $\overset{?}{w}{}^{+}_{s,nc}$ account for all the contributions due to non-conservative forces, including those due to pressure work (apart from the term mentioned in note 4, which is *now* unnecessary).

8 The Hamilton Principle, generally known as The *Least*-Action Principle, has been here renamed by making use of the adjective *Minimum* (even if less correct) only to stress the (apparent) contrast with *Maximum.* On the other hand the distinction between Minimum and *Extremum* is unessential to the establishment of the equations of motion (8.4).

9 A significantly explicative example is appropriately given by the above-mentioned "passages" (or, rather, *over-deductions*) from the traditional thermodynamic Principles to the Fourth *Thermodynamic* Principle and then, from the latter, to the Fifth Principle.