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Self-Organizing Systems, when modeled according to the Maximum Ordinality Principle, always present explicit formal solutions, in their Proper Time and Proper Space

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ABSTRACT

In a previous paper (Giannantoni 2016), we have shown that "The "Emerging Quality" of Self-Organizing Systems, when modeled according to the Maximum Ordinality Principle (MOP), offers a Radically New Perspective to Modern Science".

One of the major novelties presented in this paper is that all Self-Organizing Systems always present explicit formal solutions*. Such a very general property evidently has a huge relevance* from an operative point of view*, especially because the solutions can be obtained without any reference to "physical" forces.*

In fact the abovementioned solutions result as being deeply different from the traditional solutions adopted (or researched for) in Modern Science, precisely because they are of Ordinal Nature*. For example, when the MOP is adopted to describe the Solar System or a Galaxy, the corresponding "emerging" formal solutions suggest the abandon of the concept of "gravitation".*

In addition to the previous aspects, the same "emerging" solutions reveal another important novelty, which is particularly meaningful from a conceptual point of view: *each Self-Organizing System evolves in a "time" and a "space" which are exclusive and specific of the System analyzed. Consequently, the latter can be termed as* "Proper Time" *and* "Proper Space" of the System*.*

In order to show the profound novelty of these two concepts, the paper will present a synthetic comparison with the corresponding concepts adopted in General Relativity. A comparison that will also enable us to show an interesting "analogy": Einstein's space-time conception may substantially be seen as an attempt at introducing the formal concept of the second order "incipient derivative". At the same time, the concepts of "Proper Time" *and* "Proper Space" *also offer a "compossible" interpretation of the recent discovery of the "gravitational waves".*

INTRODUCTION

The Maximum Ordinality Principle (MOP), whose verbal enunciation asserts that "*Every System tends to maximize its Ordinality, including that of its surrounding habitat*", is formulated by means of two fundamental equations, which are so *strictly related to each other*, so as to form *a Whole* (Giannantoni 2010, 2012, 2014a,b, 2016, 2017):

The First Fundamental Equation

It is formulated as follows

$$(\underline{\tilde{d}/\tilde{d}\,t})_s^{(\tilde{m}/\tilde{n})}\{\tilde{r}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \qquad (1) \qquad (\tilde{m}/\tilde{n}) \rightarrow Max \rightarrow \{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\} \qquad (1.1)$$

where: $\{\tilde{r}\}$ is the *Relational* Space of the System under consideration, while $(\tilde{m}/\tilde{n})$ represents its corresponding Ordinality, which reaches its *maximum* when it equals $\{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\}$ (as indicated in (1.1)).

In this respect it is worth noting that:

i) the underlined symbol $(\underline{\tilde{d}/\tilde{d}\,t})_s$ explicitly indicates that the Generative *Capacity* of the System (more appropriately termed as *Generativity*), is "*internal*" to the same System, precisely because it is the one which gives origin to its Self-Organization as a Whole;

ii) the symbol "$\overset{[\rightarrow}{=}\{\tilde{0}\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact it now represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (1);

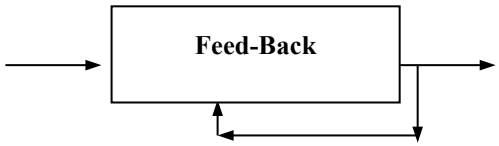
- while the symbol "$\overset{[\rightarrow}{=}$" indicates that the System, during its Generative Evolution, is persistently "adherent" to its "origin and habitat" conditions.

The Second Fundamental Equation

It is formulated as follows

$$(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\}\otimes(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\}\overset{[\rightarrow}{=}\{\tilde{0}\} \qquad (2),$$

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is *internal* to the same System, as illustrated in Fig. 1.



$$(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\}\otimes(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\}\overset{[\rightarrow}{=}\{\tilde{0}\} \quad (2).$$

Figure 1. Feed-Back between the *Relational* Space of the System and its *specific Generative Capacity*.

Equation (2), in fact, formally asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts (in the form of the product $\otimes$) with *its proper Generative Capacity* $(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$, so as to originate a *comprehensive* Generative Capacity which, *at any time*, is always adherent to the origin and habitat conditions of the Second Fundamental Equation.[1]

[1] The symbol $\otimes$ represents a generalized form of the "vector" product expressed in terms of *spinors* (see Giannantoni 2010a).

General Explicit Solution to the two fundamental Equations understood as a Whole

Equation (1) always presents an *explicit solution* which can always be written in the following general form

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\begin{pmatrix}\tilde{\alpha}_{11}(t)\\ \tilde{\alpha}_{21}(t)\\ \cdots\cdots\\ \tilde{\alpha}_{N1}(t)\end{pmatrix},\begin{pmatrix}\tilde{\alpha}_{12}(t)\\ \tilde{\alpha}_{22}(t)\\ \cdots\cdots\\ \tilde{\alpha}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\end{pmatrix}\begin{pmatrix}\tilde{\alpha}_{1N}(t)\\ \tilde{\alpha}_{2N}(t)\\ \cdots\cdots\\ \tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}} \tag{3}$$

where the *Relational* Space $\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}}$ depends on the Nature of the System analyzed, while the corresponding structure of each term of the Ordinal Matrix depends on the Specific Generativity $(\tilde{d}/\tilde{d}t)_s$.

For example, if the *Relational* Space of the System is characterized by the three topological coordinates $\{\tilde{\sigma}, \tilde{\varphi}, \tilde{\vartheta}\}$, which are always considered as *the exit of a Generative Process*, we have that

$$\{\tilde{r}\}_s = e^{\tilde{\alpha}(t)} = e^{\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\}} \tag{3.1},$$

because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\{\tilde{r}\}_s = \{\tilde{\rho}\circledR\tilde{i}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\} = \{e^{\tilde{\sigma}\circledR\tilde{i}}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\} = e^{\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\}} = e^{\tilde{\alpha}(t)} \tag{3.2}.$$

Equation (3) thus describes the *Generative Evolution* of the System as the exit of an Ordinal Cooperation of *N* Co-Productions and their associated *N* Inter-actions. At the same time, when the Process has reached its Maximum Ordinality, each term $\tilde{\alpha}_{ij}(t)$ of the Ordinal Matrix (as we will see later on) is represented by a binary-duet Relationship $\{\tilde{\alpha}_{ij}(t)\}^{\{\tilde{2}/\tilde{2}\}}$, although in the Ordinal Matrix (3) it is represented as $\tilde{\alpha}_{ij}(t)$ only for the sake of notation simplicity.

At the same time, the adoption of the brackets "{}" in Eq. (3) is explicitly finalized to remind us that the Ordinal Matrix represents a mathematical concept understood as a *Whole*.

In fact, all the elements of the Ordinal Matrix (in Eq. (3)) satisfy the following "Ordinal Relationships"

$$\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \overset{*}{=} (\sqrt[N-1]{\{\tilde{1}\}})_j \circledR \{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \tag{4}$$

for j=1,2,3,….N -1

where the additional terms $\{\tilde{\lambda}_{i,j}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$ explicitly account for the associated habitat conditions.

Eqs. (4) can also be termed as "Harmony Relationships" precisely because they show that all the elements $\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$ of the Ordinal Matrix can be obtained by means

of *one sole* arbitrary couple $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, assumed as reference, and the N-1 Ordinal Roots $(\sqrt[N-1]{\{\tilde{1}\}})_j$ of Unity $\{\tilde{1}\}$.

Consequently, if each element of the Ordinal Matrix (in Eq. (3)) is expressed in terms of the reference couple $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, the solution to Eq. (1) assumes the form

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ\left\{\begin{pmatrix}(\sqrt[N-1]{\{\tilde{1}\}})_{11}\\(\sqrt[N-1]{\{\tilde{1}\}})_{21}\\\cdots\cdots\cdots\\(\sqrt[N-1]{\{\tilde{1}\}})_{N1}\end{pmatrix},\begin{pmatrix}(\sqrt[N-1]{\{\tilde{1}\}})_{12}\\(\sqrt[N-1]{\{\tilde{1}\}})_{22}\\\cdots\cdots\cdots\\(\sqrt[N-1]{\{\tilde{1}\}})_{N2}\end{pmatrix},\begin{pmatrix}\cdots\cdots\\\cdots\cdots\\\cdots\cdots\\\cdots\cdots\end{pmatrix},\begin{pmatrix}(\sqrt[N-1]{\{\tilde{1}\}})_{1N}\\(\sqrt[N-1]{\{\tilde{1}\}})_{2N}\\\cdots\cdots\cdots\\(\sqrt[N-1]{\{\tilde{1}\}})_{NN}\end{pmatrix}\right\}} \tag{5}$$

where, always for the sake of simplicity, the term $\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}$ stands for $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$.

The same Ordinal Matrix, in addition, may be represented in a synthetic form by means one sole symbol, when adopting the following synthetic notation

$$\{(\sqrt[N-1]{\{\tilde{1}\}})_{ij}\}^{\uparrow\{\tilde{N}/\tilde{N}\}} \tag{6},$$

where the arrow "↑" explicitly reminds us that the Ordinality $\{\tilde{N}/\tilde{N}\}$ has always to be considered as being a particular form of *Over-Ordinality*.

In this way the explicit solution to Eq. (1) can more synthetically be expressed as follows

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ(\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}} \tag{7}.$$

Consequently, if such a solution is now introduced into the Global Feed-Back Process represented by Equation (2), it transforms the latter into a typical Riccati's Equation of Ordinal Nature, whose explicit solution is given by

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{B}(t)\}\circ(\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}} \tag{8},$$

where

$$\tilde{B}(t) = \left\{\begin{pmatrix}\oplus\tilde{A}(t)\\\Theta\tilde{A}(t)\end{pmatrix},\begin{pmatrix}\Theta\tilde{A}(t)\\\oplus\tilde{A}(t)\end{pmatrix}\right\} \tag{9}$$

and

$$\tilde{A}(t) = \{\{\tilde{\alpha}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}}\}\circ(\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}\}^{(\tilde{2}/\tilde{2})} \oplus \ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\}) \tag{10},$$

in which the term $\ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\})$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process.

Equation (8) then represents the Explicit "Emerging Solution" to the Maximum Ordinality Principle, formulated in two "Incipient" Differential Equations ((1) and (2)), considered as being a Whole.

General Validity of the Explicit Solution (8)

Equation (8), considered with the associated Eqs. (9) and (10), has a *general validity* because, at the same time, it is *valid* for *non-living* Systems, *Living* Systems and *Human* Systems too.

What's more, the same fact that solution (8) is *always an explicit solution* represents a very general property that evidently has a huge relevance from an *operative* point of view.

In addition, Solution (8) introduces some further fundamental novelties of *gnoseological nature* (as we will see later on), which will enable us to clearly illustrate the concept already anticipated in (Giannantoni 2016), that is: "The "Emerging Quality" of Self-Organizing Systems, when modeled according to the Maximum Ordinality Principle (MOP), offers a *Radically New Perspective to Modern Science*".

To this purpose, we can start from considering the case of *non-living* Systems.

NON-LIVING SYSTEMS

These Systems, when considered at their Maximum Ordinality, can directly be described by adopting Eq. (1), together with the associated condition (1.1), by also expressing the Relational Space of the System in the form

$$\{\tilde{r}\} = \{e^{\tilde{\alpha}(t)}\} \tag{11},$$

where $\{e^{\tilde{\alpha}(t)}\}$ is given by Eq. (3.1). So we can write

$$(\underline{\tilde{d}/\tilde{d}t})_{nl}^{(\tilde{N}/\tilde{N})}\ \{e^{\tilde{\alpha}(t)}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{12},$$

where the subscript "*nl*" stands for *non-living* (Systems), while, as already anticipated, the underlined form $(\underline{\tilde{d}/\tilde{d}t})$ of the "generator" $(\tilde{d}/\tilde{d}t)$ expressively represents the proper *Self-Organizing Capacity* of the System or, even more clearly, the *Specific Generativity* of the System analyzed.

Let us then consider, as an Ostensive Example, the case of the Solar System.

The Solar System

In this case Eq. (1) can be written in the form

$$(\underline{\tilde{d}/\tilde{d}t})_{nl}^{(\tilde{1}/\tilde{11})}\ \{e^{\tilde{\alpha}(t)}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{13},$$

where $(\underline{\tilde{d}/\tilde{d}t})_{nl} = (\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}$ (13.1) (because we assume that the System is already at its Maximum Ordinality), while the number 11 refers to the number of bodies in the Solar Systems: Sun + 9 Planets + asteroid belt.

In such a case, the corresponding Explicit Solution (8), with its associated *initial and habitat* conditions, is able to furnish some results which are extremely important *from an operative point of view,* especially because they are impossible to be obtained both in Classical Mechanics and in General Relativity. First of all the distribution of the Ordinal "*distances"* of the Planets and, in addition, their *angular anomalies* together with the *corresponding azimuthal angles of the orbital planes*.

The distribution of the Planets in the Solar System

Such a distribution, at present, is only approximately described by the semi-empirical Bode's Law, which, however, has never found a physical foundation, neither in Classical Mechanics nor in General Relativity.

This fundamentally depends on the fact that both these two Theories tend to research for a pertinent solution in mere *functional* terms and, as an immediate consequence, they have to face the formal limitations due to the famous *"Three-Body Problem"*, demonstrated as being *intrinsically unsolvable* by H. Poincaré (in 1889).

Vice versa, if the Solar System is modeled as a Self-Organizing System, that is according to Eq. (13), the Distribution of the Planets can easily be obtained by the corresponding Explicit Solution (8) (Giannantoni 2014, Gianantoni & Rossi, 2014, par. 3.2; Giannnatoni 2016, 2017 cap. 3).

In fact, the corresponding Harmony Relationships can be written as follows

$$\{\tilde{\rho}_{1,l+1},\tilde{\varphi}_{1,l+1},\tilde{\vartheta}_{1,l+1}\}_t^{\{\tilde{2}/\tilde{2}\}} = (\sqrt[11-1]{\tilde{1}})_j \circledR \{\tilde{\rho}_{12},\tilde{\varphi}_{12},\tilde{\vartheta}_{12}\}_t^{\{\tilde{2}/\tilde{2}\}} \quad \text{for } j=1,2,3,\ldots,11\text{-}1 \qquad (14)$$

where, for simplicity of notation, the *habitat conditions* are considered as being already "included" in the topological ordinal variables $\{\tilde{\rho},\tilde{\varphi},\tilde{\vartheta}\}$.

If we now assume, for example, that the couple "12" corresponds to the couple Sun-Mercury, Eqs. (14), when introduced into the Explicit Solution (8), give their fundamental contribution to the description of the *Generative Evolution* of the Solar System and, in particular, they give the distribution, at *any time*, of the Ordinal "*distances"* of the Planets (ib.).

Equations (14), in fact, together with Eq. (8), represent a *unique* "Emerging Solution" which, precisely because of its *higher information content* with respect to the initial formulation of the problem (Giannantoni 2017, ch. 1), is able to describe the "*Emerging Quality*" of the Solar System, when understood as a "Self-Organizing System".

This is also the basic reason why the distribution of the Planets, although researched for by Classical Mechanics and General Relativity, cannot be get when the Solar System is modeled in mere "functional terms".

The same considerations are also valid with reference to another problem previously anticipated. Eqs. (14), in fact, when introduced into the Explicit Solution (8), are able to solve the problem of the *azimuthal angular distribution* of the planetary orbital planes.

The azimuthal angular distribution of the planetary orbital planes with respect to the Ecliptic

As already anticipated, this phenomenon has never found a satisfactory explanation, neither in Classical Mechanics nor in General Relativity. The main reason fundamentally depends on the fact that, in the absence of any explicit solution to the "Three-body Problem", it is impossible to evaluate the exact influence between the reciprocal orbits of the Planets. The various angles of the orbital planes, in fact, are distributed in a cone of a rather large width (20°), which reduces to 10° only if the extreme Planets (Neptune and Pluto) are "excluded" (because the latter are usually considered as being rather "anomalous").

The absence of an explicit solution also depends on the fact that the Traditional Approach tends to "separate" the various phenomenological aspects, without considering the Solar System as a Whole.

Vice versa, the corresponding "Emerging Solution" to this problem is given by the same Eq. (8) that, together with Eqs. (9) and (10), describes the Solar System as a *Whole*. In fact, such an "Emerging Solution" can *contextually* be obtained together with the previous "Emerging Solution" pertaining to the Ordinal "*distances*" of the Planets (ib.).

Precessions of the Planets

The MOP is also able to describe another "Irreducible Excess" concerning the Solar System: *the Precessions of the Planets*. (ib.).

General Relativity, in fact, which has given a preliminary answer to this phenomenology, assumes that their values are always "constant" in time. In reality they result as being variable (in

time), and the Generative Evolution of their "*time variations*" is described by a pertaining *unique* "Emerging Solution" which, as usual, originates from the same Eqs. (8), (9) and (10), because these describe the Solar System as being *one sole* "Self-Organizing System".

An "Inflorescence" of Emerging Solutions

The three "Emerging Solutions" concerning the Ordinal "*distances"* of the Planets, the *azimuthal angles of their orbital planes* and the correlative *angular anomalies*, do not represent "three *independent* solutions", because all of them "emerge" from the *same formulation* of the Maximum Ordinality Principle (Eq. (13)) and from the same Explicit Solution (given by Eqs. (8), (9) and (10)), when the latter is explicitly referred to the Solar Systems (ib.).

Consequently, we can say that the three abovementioned Solutions represent *one sole* and *unique* "*Inflorescence*" of "Emerging Solutions", which is a clear "sign" of the fact that the Solar System is a *unique and sole* "Self-Organizing System".

Systems and sub-Systems

The Ostensive Example of the Solar System is also able to show that, in an Ordinal Perspective, such as the one suggested by the MOP, we cannot speak about "Systems and sub-Systems".

The concepts of "Systems" and "sub-Systems", in reality, are perfectly conform to a Perspective in which the reciprocal relationships between the elements of a "System" are systematically expressed in terms of a "*distance*".

The concept of "*distance*", in fact, tends to *divide*, more than to *unify*. The same etymology of the word (from Latin "*dis-stant*") indicates that "one element *stays here* and the other one *stays there*" or, equivalently, "*one is here and the other one is there*".

In an Ordinal Perspective, on the contrary, the term "distance" should be replaced by a different term, possibly able to indicate the "*union*" of two (or more) elements.

In this respect, by introducing a *neologism* (that "rhymes" with the term "distance", but it exactly indicates the opposite meaning), we could say that the same value that in a "functional" Perspective represents a "distance", in an Ordinal Perspective indicates a "*uni-ance*". That is, it indicates that the two elements form "one sole thing".

In fact, when a considered value is introduced into the Harmony Relationships (obviously together with all the other "*uni-ances*"), it reveals that the System is a *Whole of Ordinal Nature*, in perfect adherence with the Maximum Ordinality Principle.

This consequently means that, if we "trace" an "ideal closed surface" to "isolate" a limited number of elements of the System (for example, the first three Planets of the Solar System), this procedure is able to define a "sub-System" only in the contest of a "functional" Perspective.

The same procedure, on the contrary, when considered in the contest of an Ordinal Perspective, radically "alters" the "*uni-ances*" between the various elements of the System, by "reducing" them to simple cardinal "*distances*", and by contextually denying, in this way, the concept of the System as an *Ordinal Whole*.

Absence of "gravitation" forces

Another important novelty, both at a conceptual and practical level, is that the previous description of the Solar System, based on the MOP, can be obtained without *any reference* to "gravitation forces".

The latter in fact establish some specific "functional relationships" between two (or more) Planets, while the MOP does not contemplate "functional relationships", but only Ordinal Relationships (Giannantoni 2012, 2014a,b, 2016).

In fact, the Emerging Solutions obtained from the MOP describe the Solar System in such a way as the various Planets are *related to each other* according to *Ordinal Relationships*. In other words, according to the same typology of "relationships" which precisely take origin from any *generative processes*, such as, for example, the *genesis of two* (*or more*) *brothers*.

"Brothers", in fact, are properly defined as such *not* because of their *direct* relationships. That is, because they respect each other or they love each (in fact, they might also hate each other). They are "brothers", *in essence*, because generated by the same father (or the same mother, or both). That is, because of their *direct relationship* with the *generative cause* of their being born.

In the case of the Solar System, the various Planets are related to each other in terms of *genetic* relationships, precisely those relationships that take origin from the same Specific Generativity of the Solar System, which represents something which is *unique*, *specific* and *irreducible*. Consequently, in spite of their reciprocal distinction, the various Planets must be considered as *one sole entity*. That is, as a Whole. This is because, as already said, the single Planets refer to a *unique and sole genetic principle* (characteristic of each given System), which is also the basis of *their specific Ordinal Relationships* between them.

In accordance with such a concept, the *direct* relationship between *any* two Planets can be considered as being of the *second order*, with respect to the Fundamental Relationship they have with the pertinent Genetic Principle. Nonetheless, such Relationships are always of *Ordinal Nature*. In fact, the Ordinality of the Relationship between *each couple* of Planets, according to Eqs. (8) and (14), is equal to $\{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{2}/\tilde{2}\}$.

This is also the reason why the Explicit Solutions that "emerge" from the MOP contemporaneously give the distribution of all the Planets with respect to *any couple* of planetary bodies assumed as reference.

The hypothesis of "gravitation", on the contrary, "reduces" the Ordinal Relationship between two (or more) Planets to a "direct" *cardinal* relationship, because it is understood as an "efficient causality" of "functional nature".

Such hypothetical assumptions are then precisely those that lead to the famous *"Three-Body Problem"*, demonstrated as being *intrinsically unsolvable* by H. Poincaré (in 1889).

General validity of Eq. (12) for *all* non-living Systems

The mathematical formulation of the MOP according to Eq. (12), previously illustrated by means of the Ostensive Example of the Solar System, is always valid for *any* non-living System.

This is because all non-living Systems *always have the same* Specific Generativity (13.1). The only difference between the various Systems relies in the number of components (N) and their associated *origin and habitat* conditions ($\{\tilde{0}\}$), which characterize, in a specific way, their corresponding Explicit Solutions, always given by Eq. (8), together with Eqs. (9) and (10).

LIVING SYSTEMS

Living Systems differ from non-living Systems because of their own Specific Generativity.

In fact they can always be described on the basis of Eqs. (1) and (1.1), when the symbol $(\underline{\tilde{d}/\tilde{d}t})_s$ now represents their specific Generativity $(\underline{\tilde{d}/\tilde{d}t})_{ls}$, that is

$$(\underline{\tilde{d}/\tilde{d}t})_{ls}^{(\tilde{m}/\tilde{n})}\{e^{\tilde{\alpha}(t)}\}^{[\rightarrow} = \{\tilde{0}\} \qquad (15),$$

where the subscript "*ls*" stands for "living systems", and the explicit expression of the Specific Generativity is given by the following exponential form

$$(\underline{\tilde{d}/\tilde{d}t})_{ls} = e^{(\tilde{d}/\tilde{d}t)^{\{\tilde{2}/\tilde{2}\}}} - 1 = \oplus(\tilde{D}/\tilde{D}t) \oplus \frac{1}{2!}(\tilde{D}/\tilde{D}t)^{\tilde{2}} \oplus \frac{1}{3!}(\tilde{D}/\tilde{D}t)^{\tilde{3}} \oplus ..... \qquad (16)$$

where $(\tilde{D}/\tilde{D}t)$ is adopted for a more synthetic representation of $(\tilde{d}/\tilde{d}t)^{\{\tilde{2}/\tilde{2}\}}$.

Such an expression can be assumed as being representative of the Specific Generativity of a Living System because:

i) the number "*e*" is very often understood as a "figure" of a Generative Process in the same Modern Science;
ii) what's more, the expansion series (16) is now understood in *Ordinal terms* (as indicated by the various "tilde" notations) and, in such a form, accounts for the "*net" Emerging Quality* from a Generative Process;
iii) in addition, the various terms which appear in the right hand-side of Eq. (16) are not simply "added" between them (as it happens in the case of a traditional cardinal expansion series), but they are thought of giving origin to a "*Composition*" ($\oplus$) of *Ordinal Nature*, which is thus understood as a Whole;
iv) while, at the same time, the number of the terms of the Composition depends on the specific System analyzed. This is why Eq. (16) can be rewritten as follows

$$(\underline{\tilde{d}/\tilde{d}\,\tilde{t}})_{ls} = Exp_n(\tilde{D}/\tilde{D}\,\tilde{t}) = \oplus(\tilde{D}/\tilde{D}\,\tilde{t}) \oplus \frac{1}{2!}(\tilde{D}/\tilde{D}\,\tilde{t})^{\tilde{2}} \oplus \frac{1}{3!}(\tilde{D}/\tilde{D}\,\tilde{t})^{\tilde{3}} \oplus .... \qquad (17),$$

where the subscript n , in the symbol Exp_n , indicates *the number of terms considered*.

The case of one sole Exponential

We may start from considering the formulation of the MOP for a Living System, whose Specific Generativity is characterized by a single Exponential, that is

$$\{Exp_n(\tilde{D}/\tilde{D}\,\tilde{t})\}^{(\tilde{N}/\tilde{N})}\ \{e^{\tilde{\alpha}(t)}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \qquad (18).$$

Such a formulation can be adopted, for example, to describe the Ordinal configuration of a single Protein, while the number $\tilde{N}$ depends on the number of Amino Acids of the same Protein, and the number n is strictly related to the accuracy desired in the description.

The case of one sole Exponential in the form Exp_2

Eq. (18), written with one sole Exponential in the form Exp_2, that is

$$Exp_2 = \oplus(\tilde{D}/\tilde{D}\,\tilde{t}) \oplus \frac{1}{2!}(\tilde{D}/\tilde{D}\,\tilde{t})^{\tilde{2}} \qquad (19)$$

was adopted to analyze the Ordinal Interaction between two different Proteins, considered at their Maximum Ordinality, in order to study possible improvements of some pharmaceutical treatments (Giannantoni 2015).

The example analyzed refers to *diabetic therapy*. It is well-known in fact that human Insulin has a reduced affinity with blood Albumin, so that the subcutaneously injected Insulin cannot efficiently be conveyed by blood Albumin in the various parts of the body.

The therapy then consists in adopting a modified form of Insulin, which presents a higher affinity with blood Albumin. The modified form of Insulin usually adopted is Insulin detemir, also termed as Levemir.

Figure 2 represents the three-dimensional structure of human Insulin (51 Amino Acids), obtained by means of an appropriate simulator, termed as EQS (Emerging Quality Simulator), based on the MOP (ib.). Such a Figure was obtained in less than 1 sec, by running the Simulator on a simple PC (10^9 Flops). (ib.)

The same was done for Albumin (Fig. 3) and, immediately after, the same EQS was adopted to represent the topological structure of the *final compound* emerging from their Ordinal Interaction.

The same analysis was successively repeated by considering the modified form of Insulin (Levemir) and its Ordinal Interaction with Albumin, in order to show the corresponding therapeutic improvements (ib.).

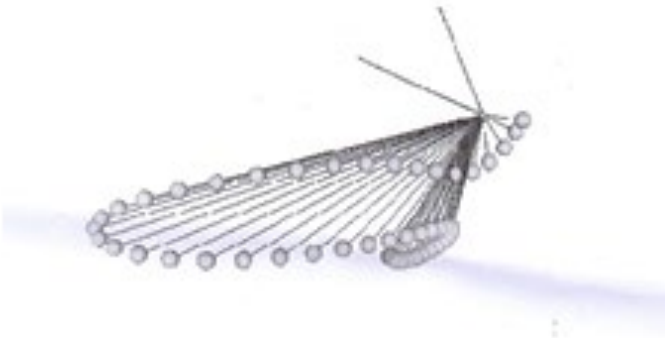


Figure 2. *Three-dimensional structure of human Insulin (51 Amino Acids: 21 in subunit A and 30 in subunit B)*

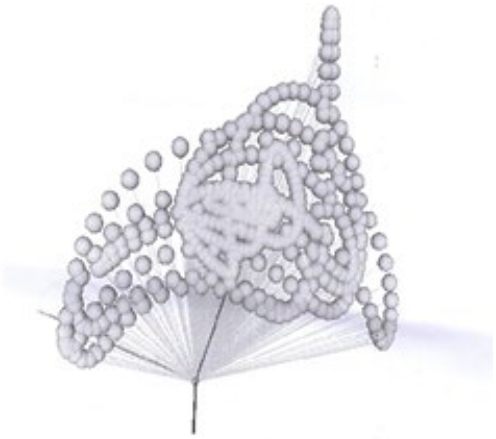


Figure 3. *Three-dimensional structure of blood Albumin (585 Amino Acids).*

The case of two Exponentials

The representation of the Specific Generativity of a Living System in terms of two Exponentials, that is

$$(\underline{\tilde{d}/\tilde{d}\,t})_{ls} = \{\tilde{E}\,xp_{n_1}^{\{\tilde{N}_1/\tilde{N}_1\}}\{\tilde{E}\,xp_{n_2}^{\{\tilde{N}_2/\tilde{N}_2\}}(\tilde{D}/\tilde{D}\,t)\}\} \qquad (20)$$

can be adopted to study, for example, a single Protein together with its N_1 amino acids. That is:
i) to study its "secondary structure" (as usually termed in Biology, and here modeled by the first Exponential);
ii) together with its "tertiary structure", that is the topological Ordinal Structure of the various atoms of each Amino Acid (whose maximum number is assumed equal to N_2), and here modeled by means of the second Exponential.

In such a case the Explicit Solution (8) modifies in the sense that:

- each element $\tilde{\alpha}_{i,j}(t)$ of the main Ordinal Matrix in Eqs. (3) and (5) transforms, in turn, into a new Ordinal Matrix
- so that the Ordinal Matrix in Eqs. (3) and (5) assumes a structure that is similar to an Ordinal "Matryoshka", that is an Ordinal Matrix made up of Ordinal Matrices;
- while the specific properties of each element are always defined by means of the associated origin and habitat conditions.

The case of more than two Exponentials

The previous case suggests that, in principle, there might be some Living Systems characterized by a higher number of Exponentials.

In the case of *three* Exponential the Specific Generativity can be written as

$$(\underline{\tilde{d}/\tilde{d}\,t})_L = \{\tilde{E}\,xp_{n_1}^{\{\tilde{N}_1/\tilde{N}_1\}}\{\tilde{E}\,xp_{n_{21}}^{\{\tilde{N}_2/\tilde{N}_2\}}\{\tilde{E}\,xp_{n_3}^{\{\tilde{N}_3/\tilde{N}_3\}}(\tilde{D}/\tilde{D}t)\}\}\} \qquad (21).$$

Such an equation, however, can preferably be rewritten by adopting the notation "$\uparrow$"

$$(\underline{\tilde{d}/\tilde{d}\,t})_L = \{\tilde{E}\,xp_{n_1}^{\{\tilde{N}_1/\tilde{N}_1\}} \uparrow \tilde{E}\,xp_{n_{21}}^{\{\tilde{N}_2/\tilde{N}_2\}} \uparrow \tilde{E}\,xp_{n_3}^{\{\tilde{N}_3/\tilde{N}_3\}}(\tilde{D}/\tilde{D}t)\} \qquad (22),$$

in order to explicitly *point out* that the various Exponentials are not related to each other in "functional" terms (as usually happens in the Traditional Differential Calculus), but only in "Ordinal" terms.

This case also suggests that there might be some Living Systems that could be modeled by considering *more than three* Exponentials. The same Human Organism, in fact, in principle could be described by means an appropriate number of Exponentials. This correspondently means that the Explicit Solution (8) transforms in such a way as each element $\tilde{\alpha}_{i,j}(t)$ in the main Ordinal Matrix becomes a sequence of successive "Matryoshkas of Matryoshkas".

In this respect it is also worth pointing out that the Ordinality of the first Exponential $\tilde{E}\,xp_{n_1}^{\{\tilde{N}_1/\tilde{N}_1\}}$ is equal to $\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}_1/\tilde{N}_1\}$. The first Exponential, in fact, can always be rewritten in its "basic form" (see Eq. (16)) as follows

$$\tilde{E}\,xp_{n1}^{\{\tilde{N}_1/\tilde{N}_1\}} = \{\oplus(\tilde{d}/\tilde{d}\,t)\oplus\frac{1}{2!}(\tilde{d}/\tilde{d}\,t)^{\{\tilde{2}/\tilde{2}\}}\oplus\frac{1}{3!}(\tilde{d}/\tilde{d}\,t)^{\{\tilde{2}/\tilde{2}\}\uparrow\tilde{2}}\oplus....\}^{\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}_1/\tilde{N}_1\}} \qquad (22.1).$$

Eq. (22.1) then clearly shows that, when the Specific Generativity of the System is represented by *one sole* Exp, the Maximum Ordinality of the System (see Eq. (1.1)) is equal to $\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}_1/\tilde{N}_1\}$, which, however, is now properly referred to the basic Generativity which appears in Eq. (22.1)

$$\{\oplus(\tilde{d}/\tilde{d}\,t)\oplus\frac{1}{2!}(\tilde{d}/\tilde{d}\,t)^{\{\tilde{2}/\tilde{2}\}}\oplus\frac{1}{3!}(\tilde{d}/\tilde{d}\,t)^{\{\tilde{2}/\tilde{2}\}\uparrow\tilde{2}}\oplus....\} \qquad (22.2).$$

Analogously, in the case of a System whose Specific Generativity is represented by more than one Exp, for example by Eq. (22), the Maximum Ordinality of the System (see Eq. (1.1)), is given by

$$\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}/\tilde{N}\}\overset{*}{=}\{\tilde{2}/\tilde{2}\}\uparrow\{\{\tilde{N}_1/\tilde{N}_1\}\uparrow\{\tilde{N}_2/\tilde{N}_2\}\uparrow\{\tilde{N}_3/\tilde{N}_3\}\} \qquad (22.3),$$

while the *basic* Generative terms of $\tilde{E}\,xp_{n1}^{\{\tilde{N}_1/\tilde{N}_1\}}$ (see Eq. (22.2)), together with those corresponding to $\tilde{E}\,xp_{n_{21}}^{\{\tilde{N}_2/\tilde{N}_2\}}$ and $\tilde{E}\,xp_{n_3}^{\{\tilde{N}_3/\tilde{N}_3\}}$, contribute to the genesis of the *Ordo-cardinality* of the *single elements* $\tilde{\alpha}_{i,j}(t)$ of the various "Matryoshkas of Matryoshkas".

Systems and sub-Systems

As we have already seen, the Specific Generativity of Living Systems can be represented by means a variable number of Exponentials. The latter, however, as indicated by Eq. (22), cannot be considered as being separated from each other, because they always represent a *unique and sole* Generativity of Ordinal Nature. Consequently, in analogy to the case of non-living Systems, we cannot speak about "Systems and sub-Systems".

Nonetheless, this fact does not prevent from studying the behavior of a given set of elements of a Living System.

In this case, however, such a set of "selected" elements has always to be studied in the context of a *unique and sole* Ordinal System. That is, a System which is always understood as a *Whole* and, consequently, as being *not separable* in "parts" (otherwise the latter will be described and analyzed in mere "functional terms").

In spite of such an "analogy" between Living Systems and non-living Systems, due to the absence of distinction between "Systems and sub-Systems", the former present many deep differences with respect to the latter. One of the major differences is that, in the case of Living Systems, we can speak about a "*Hierarchy* among Living Systems".

Hierarchy among Living Systems

The same fact that the Specific Generativity of Living Systems can be represented by means of a differentiated number of Exponentials suggests that Living Systems, when considered as *one sole Class*, are characterized by *a proper internal Hierarchy*.

This is clearly shown by the same definition (22.3), which is precisely based on the Ordinality of the various Exponentials that represent the Specific Generativity of each considered System.

Such a concept of "Hierarchy" may also have a huge relevance from a *gnoseological* point of view. The term "*Life*", in fact, may assume a *different meaning* according to the number of Exponentials (each one characterized by a specific Ordinality), which are required to represent the Specific Generativity of each given Living System.

Absence of "electrostatic" forces

In analogy to the basic considerations developed in the case of non-living Systems (see par. 2.3), the MOP offers an important novelty to Modern Science with reference to Living Systems.

In fact, the previous descriptions of Proteins and Amino Acids can be obtained without any reference to "electrostatic forces", such as Coulomb force, Van der Waals forces, Hydrogen Bonds, etc.

This represents a *general concept* which is always valid for *any other* Living System too.

"Electrostatic forces", in fact, establish some specific "functional relationships" between two (or more) components of any given Living System, while the MOP does not contemplate any form of "functional relationships", but only Ordinal Relationships.

In fact, the "Emerging Solutions" obtained by means of the MOP describe each Living System as a Whole, in which the various "components" are *related to each other* according to *Ordinal Relationships* of *Generative Nature*.

In other words, as already anticipated at par. 2.3, such Ordinal "Relationships" are similar to those that take origin from *one sole generative process*, as previously illustrated in the case of the *genesis of two or more brothers*.

Vice versa, the hypothesis of "electrostatic forces" *reduces* the Ordinal Relationship between two (or more) components of a Living System to a "direct" *cardinal* relationship, because those "forces" are always an expression of an "efficient causality" of "functional nature".

On the other hand, this is precisely the hypothesis that leads to the unsolvability and intractability of both *Protein Folding* and *Protein-Protein Interaction*. Once again, as a consequence of *"Three-Body Problem"*, when the latter is considered with reference to the Amino Acids of the various Proteins (Giannantoni 2015).

HUMAN SYSTEMS

Such an "acceptation" wants to indicate either "Systems *made up of men*" (even a *single* man) or "Systems *realized by men*". The corresponding formulation of the MOP can correspondently be formulated as follows

$$(\tilde{d}/\tilde{d}\,\tilde{t})_{HS}^{(\tilde{m}/\tilde{n})}\{e^{\tilde{\alpha}(t)}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{23},$$

where the Specific Generativity can still be represented as a sequence of Exponentials similar to Eq. (22), because the considered Systems are still "Living Systems", even if the *fundamental difference* now consists in the fact that:

i) the *Relational Space* is *not* characterized by the topological variables $\{\tilde{\sigma}, \tilde{\varphi}, \tilde{\vartheta}\}$ such as in the previous cases;

ii) because the exponent $\tilde{\alpha}(t)$ (see Eq. (3.1)) assumes a different Ordinal Structure according to the Human System each time considered.

In fact, *at the most general level*, $\tilde{\alpha}(t)$ is characterized by "variables" that represent (especially in the case of a *single* man) the Generative Modalities of *Thinking*, *Decision Making* and *Acting* (or, equivalently, *Logic*, *Will* and *Acts*), considered as being *Activities of Ordinal Nature*, such as, for instance, the *Generative Formal Logic* (Giannantoni & Rossi, 2014; Giannantoni 2017).

Such "variables", in fact, are directly referable to the Fundamental Presuppositions of the MOP and its corresponding Ordinal Perspective based on the "Emerging Quality" of Self-Organizing Systems (Giannantoni 2016).

This means that they are *always* "*subjacent*" to the description of *any* Human System, even when the System analyzed suggests the adoption of a Relational Space characterized by more specific variables.

The Specific Generativity, in fact, when considered *at its most general level*, *represents an intrinsic property of the Human System*, which is able to transform the System into a Self-Organizing System understood as Whole.

In this respect it is worth noting that, in spite of the wide variety of "Human Systems" that can be considered (such as Constitutional Systems, Juridical Systems, Economic Systems, etc.), the concept of "Human System" is generally referred to a *single* Society or a single State. In particular with specific reference to its Constitution, as well as to its pertinent Economic and/or Energetic-Environmental Systems.

After such due fundamental premises, we will now consider the case of a *particular set of States.*

The "Human System" represented by United Europe (UE)

Such an Ostensive Example will be analyzed in the light of an *Ordinal Perspective*, which results as being "com-possible" with respect to the traditional "functional" perspectives. In fact, the 28 States that constitute the UE (United Europe) in reality are "united" between them only by means of international treaties. The different States, in fact, are still profoundly different from each other for many aspects, especially for their pertinent economies.

Consequently, as an Ostensive Example of a "Human System", we will analyze the Economic System of UE, *when the latter is understood as a Whole*, in order to describe the possible evolution of the present Economic System toward its Maximum Ordinality.

In this case the MOP can be formulates as indicated by Eq. (23), where the Relational Space of the System can be written in the form $\{\tilde{e}^{\tilde{\alpha}(t)}\} = e^{\{\ln \tilde{K} \tilde{\otimes} \tilde{i} \tilde{\oplus} \tilde{L} \tilde{\otimes} \tilde{j} \tilde{\oplus} \tilde{N} \tilde{\otimes} \tilde{k}\}}$, because it is now characterized by three new variables, such as $\{\tilde{K},\ \tilde{L},\ \tilde{N}\}$, where $\tilde{K}$ = Capital, $\tilde{L}$ = Labour, $\tilde{N}$ = Natural Resources (Giannantoni 2011). These variables in fact represent a "direct reflex" of the way of *Thinking*, *Decision Making* and *Acting*, precisely because they are now considered as being of *Generative Nature* and, at the same time, as representing *one sole* Ordinal Entity.

Such *Ordinal Variables*, when considered with reference to any given State (*i*), can respectively be assumed as being equal to $\tilde{K}_i = GDP_i$; $\tilde{L}_i$ = occupied Workers; $\tilde{N}_i$ = Natural Resources of the same State (*i*) (at the net of the import/export quantities because, by definition, the latter are already accounted for by GDP_i).

The same variables, vice versa, when considered according to the traditional economic analysis, are usually thought of as mere "functional" variables, because they are considered with the sole reference to their specific corresponding cardinal values.

In our case, on the contrary, when the Specific Generativity of UE is structured as an appropriate sequence of Exponentials, it is able to describe both the Generativity that is subjacent to the Ordinal organization of the UE and, at the same time, the different internal and specific organizations of the single States.

In fact, if Eq. (23) is characterized by one sole Exponential, it is able to describe the Ordinal Organization of the 28 States understood as a Whole. In the case of two Exponentials, the same Eq. (23) is able to describe, in addition, the Ordinal Organizations of the various Regions internal to each State. While in the case of three Exponentials it is also able to describe the Ordinal Organization of the various Provinces of each single Region and, at the same time, the entire System always understood as a Whole.

For the sake of simplicity, we will now consider the sole Ordinal Organization of the 28 States. This means that the Specific Generativity in Eq. (23) is represented by one sole Exponential, and the MOP is thus formulated on the basis of the following Generativity

$$(\tilde{\underline{d}}/\tilde{d}\,t)_{HS} = \{\tilde{E}\,xp_{n_1}^{\{\tilde{N}_1/\tilde{N}_1\}}\} \qquad (24),$$

where $\tilde{N}_1 = 28$ and $n_1 = 2$, in order to have, in this way, a more accurate description.

The solution to the MOP based on the Specific Generativity (24) is always given by Eq. (8), while the Fundamental Harmony Relationships (associated to Eq. (8)) are now expressed by (see Giannantoni 2009, 2011)

$$\{\tilde{K}_{1,j+1}, \tilde{L}_{1,j+1}, \tilde{N}_{1,j+1}\} \overset{*}{=} (\sqrt[N-1]{\{\tilde{1}\}})_j \,\tilde{\otimes}\, \{\tilde{K}_{12}, \tilde{L}_{12}, \tilde{N}_{12}\} \qquad \text{for } j = 1,3,.....(28-1) \qquad (25).$$

Such Harmony Relationships are similar to those presented in the case of a single Protein, with the fundamental difference, however, that now the explicit solution describes the *Generative Evolution* of the 28 States of UE at a macro-economic level, when such an "Evolution" is understood as an "Ordinal Trend" toward the Maximum Ordinality. This is because the Specific Generativity is able to "guide" the various States *toward* the genesis of a Self-Organizing System, understood as a Whole. Such an Objective, however, is actually reached *only if* the various States are really willing to actuate that "operative route" explicitly indicated by the MOP, in the respect of their specific way of Thinking, Decision Making and Acting at the level of their pertinent Parliaments.

The actual progressive *trend* toward the Maximum Ordinality can easily be "recognized" by means a *systematic comparison* between the 28 triplets $\{\tilde{K}_i, \tilde{L}_i, \tilde{N}_i\}$ which "emerge" from the MOP, and the *actual* values of the 28 corresponding triplets that, at any time, characterize the present status of UE.

In this way it is possible to recognize the corresponding specific modalities according to which the System is progressively "*tending*" toward its Maximum Ordinality.

Systems and sub-Systems

Also in this case, if we are in the presence of a Human System that has actually become a real Whole, it is impossible to speak of "Systems and sub Systems". In fact, as previously seen in the case of non-living Systems and Living Systems too, any "isolation" of a portion of the System "brakes" its unity as *One Sole* Ordinal System, which consequently will no more be characterized by its own internal Harmony Relationships.

"PROPER TIME" AND "PROPER SPACE" AS "EMERGING PROPERTIES" OF EACH SELF-ORGANIZING SYSTEM

In addition to the previous aspect of Explicit Solutions, which surely has a huge relevance from an *operative* point of view, we will now consider another aspect, which, vice versa, is particularly meaningful from a *conceptual* or, even better, *gnoseological* point of view. That is, the fact that *each Self-Organizing System evolves in a "time" and a "space" which are exclusive and specific of each considered System*. Consequently, the latter can be termed as "*Proper Time*" and "*Proper Space*" of the System analyzed.

In fact, if we consider the Emerging Solution (8), we can easily recognize that in the expression of $\tilde{A}(t)$ (see Eq. (10)) there is the term $\ln\{\tilde{c}_1 \oplus \{\tilde{c}_2, t\}\}$, where the "time *t*" appears as a member of an *Ordinal Duet*, which consequently indicates an "Irreducible Excess" of Ordinal Nature with respect the simple "cardinal" product $(c_2 \cdot t)$.

This clearly means that the "time *t*" does not flow in a "uniform way", as usually it is supposed by both Physics and Biology but, at "any time *t*", the contribution $\{\tilde{c}_2, t\}$ to the Evolution of the System represents a *completely new* "entity". In fact, if we consider the contribution of $\{\tilde{c}_2, t\}$ at the time $(t + \Delta t)$, that is $\{\tilde{c}_2, (t + \Delta t)\}$, this term results as being totally "irreducible" to the contribution corresponding to the previous time *t*.

This is equivalent to say that: "*each time represents a new Origin*".

Consequently, when such an "irreducible term" $\{\tilde{c}_2, t\}$ "contributes" to the General Solution (8), it allows us to assert that: "*each Self-Organizing System evolves in a "time" which is exclusive and specific of the considered System analyzed*".

As a further immediate consequence, it is easy to recognize that the same happens with reference to the *Ordinal Space* of the same System. This is because the General Solution (14) represents, in Ordinal Terms, the Generative Evolution of the "*Topological Configuration*" of the considered System.

Consequently, the same "irreducible term" $\{\tilde{c}_2, t\}$ also gives its specific and "*irreducible*" contribution to the Ordinal Topological Reconfiguration of the System, always described by the same Eq. (8). This also means that the *new Topological Reconfiguration* of the System, at "*any time* $(t + \Delta t)$, results as being *always irreducible* to the Topological Configuration of the System considered at the "*time t*".

Such considerations concerning the two aspects previously analyzed can be synthesized by asserting that: *each Self-Organizing System evolves in a "time" and a "space" which are exclusive and specific of each System analyzed. Consequently, the latter can be termed as* "Proper Time" *and* "Proper Space" of the System.

The concept of *space-time* in General Relativity vs *Proper Space* and *Proper Time* in the light of the MOP

As is well known General Relativity introduces the concept of "*space contraction*" and "*time dilatation*" between two reference systems in a reciprocal movement, according to the following relationships (Landau 1966, p. 21)

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - V^2/c^2}} \qquad (26) \qquad\qquad \Delta x' = \Delta x \cdot \sqrt{1 - V^2/c^2} \qquad (27).$$

Let us then consider the motion of a body represented by the function $x(t)$ with respect to a basic reference system, while the same motion is described by $x'(t')$ in a second reference system that is in movement with respect to the first one.

On the basis of Eqs. (26) and (27) it is possible to compare the *acceleration* of the body whose motion is described by $x(t)$, with the second order "*incipient*" derivative of the same motion, now represented by $\tilde{x}(t^*)$ because the motion is now described according to an *Ordinal Perspective*.

By considering that

$$x(t) = \gamma \cdot x(t') = \gamma \cdot e^{\alpha(t')} \qquad (28), \qquad \text{where} \qquad \gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \qquad (29),$$

as a first step of our analysis we can suppose that the acceleration $\frac{d^2}{dt^2}x(t)$ might be evaluated as $\frac{d^2}{dt'^2}\gamma \cdot e^{\alpha(t')}$, that is with reference to the time *t'*, which is the time characteristic of the reference system considered in motion. So we have

$$\frac{d^2}{dt'^2}\gamma \cdot e^{\alpha(t')} = \gamma \cdot e^{\alpha(t)} \cdot \{[\alpha'(t')]^2 + \alpha''(t')\} \qquad (30).$$

Such a result can now be compared with the *second order "incipient" derivative* of the Ordinal Relationship $\tilde{x}(t^*)$

$$\left(\frac{\tilde{d}}{\tilde{d}\,t^*}\right)^{\tilde{2}} e^{\tilde{\alpha}(t^*)} = e^{\tilde{\alpha}(t^*)} \circ \{\overset{\circ}{\tilde{\alpha}}(t^*)\}^{\tilde{2}} \qquad (31)$$

where, for the sake of clearness, the time corresponding to the Ordinal Description $x(t^*)$ is represented by t^*.

The researched comparison, however, can actually be done only after having "reduced" the Ordinality of the Relationship (31) to its corresponding "reflexed" cardinality.

In order to do that, the Ordinality $\tilde{2}$ in Eq. (31) has to be "reduced" to a simple *square* 2, that is to a simple product $\{\overset{\circ}{\tilde{\alpha}}(t^*) \cdot \overset{\circ}{\tilde{\alpha}}(t^*)\}$, which evidently accounts for the sole "*reflexed*" cardinalities. In such a "reduced" form we have

$$\left(\frac{\tilde{d}}{\tilde{d}\,t^*}\right)^{\tilde{2}} e^{\tilde{\alpha}(t^*)} = \downarrow e^{\tilde{\alpha}(t^*)} \cdot \gamma_1 \cdot \{\overset{\circ}{\tilde{\alpha}}(t^*) \cdot \overset{\circ}{\tilde{\alpha}}(t^*)\} \qquad (32),$$

where the "arrow ↓" reminds us the previous *reduction process*, while γ_1 represents a "compensation factor" of the lower value of the "reflexed" cardinality in the passage from the Ordinality $\tilde{2}$ to a simple square 2.

Under such conditions, the second hand side of Eq. (32) can be considered as being "equal" (only by "assignation" $(\overset{*}{=})$), to the second hand side of Eq. (30), when the term $\{[\alpha'(t')]^2 + \alpha''(t')\}$ "equals" the term $\{\overset{\circ}{\tilde{\alpha}}(t^*) \cdot \overset{\circ}{\tilde{\alpha}}(t^*)\}$ and, at the same time, the factor γ is comparable with the "compensation factor γ_1".

More in general, the comparison (always considered as being valid by "assignation") can be written as follows

$$\gamma \cdot e^{\alpha(t')} \cdot \{[\alpha'(t')]^2 + \alpha''(t')\} \overset{*}{=} \downarrow e^{\alpha(t^*)} \cdot \gamma_1 \cdot \{\overset{\circ}{\tilde{\alpha}}(t^*) \cdot \overset{\circ}{\tilde{\alpha}}(t^*)\} \qquad (33).$$

At this stage, for a more appropriate comparison, we have to take into account that the first hand side of Eq. (33) is formulated in the time *t'*, while the second side of the same Eq. (33) is formulated in terms of the time t^*, which represents the *Proper Time* corresponding to the Ordinal Description of the motion $x(t^*)$.

Consequently, if we consider that $(dt')^2 = (dt)^2 \cdot (\sqrt{1 - V^2/c^2})^2$, there is a new factor $\gamma_2 = (\sqrt{1 - V^2/c^2})^2$ that appears at the denominator of the first hand side of Eq. (33). At the same time, a new "compensation factor" will appear at the second hand side of the same Eq. (33), as a consequence of the "reduction" of the Ordinality of the Proper Time of the Ordinal System to a reflexed "*scalar*" *time*.

The term $\{\tilde{c}_2, t\}$, in fact, which appears in the Emerging Solution (Eq. (8)) and now formally represented as $\{\tilde{c}_2, t^*\}$, can be transformed into its "*reflexed*" *cardinality* by considering the following successive "reductions":

$$\{\tilde{c}_2, t^*\} = \downarrow \{\tilde{c}_2 \circ t^*\} = \downarrow (c_2 \cdot t^*) \qquad (33.1).$$

Such a double reduction of Ordinality, operated to get the corresponding cardinality, can be accounted for by another "compensation factor" γ_3, that can be thought of being structured in the form of the scalar product $\gamma_3 \cdot t^*$.

Under such conditions, the *equivalence* (always "by assignation") between the two sides of Eq. (33) can more appropriately be written as

$$(\gamma / \gamma_2) \cdot e^{\alpha(t)} \cdot \{[\alpha'(t')]^2 + \alpha''(t')\} \overset{*}{=} \downarrow e^{\alpha(t^*)} \cdot \gamma_1 \cdot \gamma_3 \cdot \{\overset{\circ}{\alpha}(t^*) \cdot \overset{\circ}{\alpha}(t^*)\} \qquad (34),$$

and it can be considered as being satisfied when the factor $1/\gamma_2$ is representative of the "compensation factor γ_3".

The previous considerations should then be able to show that Einstein's "*space-time conception*" represents a *particular modality* at introducing the concept of the *second order* "*incipient*" *derivative* (see also Giannantoni 2007). Such a particular modality, however, manifests itself at *a simple cardinal level*, as a consequence of the "reduction" process of the *Proper Space* and *Proper Time* of a given System. This means that Einstein's *space-time conception* in reality corresponds to the introduction of the *second order* "*incipient*" *derivative* considered at its mere "*cardinal level*".

"Gravitational waves" in the light of the Proper Time and Proper Space

The concepts of "*Proper Time*" and "*Proper Space*" of each Self-Organizing System offer the possibility of a different (albeit com-possible) interpretation of the recent discovery of the (so-called) "gravitational waves".

The latter in fact, whose "discovery" has been recently recognized by a Nobel Prize (October 2017) as a consequence of their experimental "revelation", are usually understood as "*physical*" waves, which are produced by a strong "shock" between two very huge celestial bodies, such as two Black Holes.

The enormous quantity of energy released during such an extremely rare event, produces some consequential "deformations" of Einstein's *space-time*. "Deformations" that manifest themselves in the form of "gravitational waves", which can be revealed even at a very huge distance, by means of appropriate instrumentations. In particular, they can be reveled even on Earth, as it really happened in the experiments performed during last two years.

However, when the "gravitational waves" are reconsidered in the light of the concepts of "Proper Time" and "Proper Space", they are no more understood as being "physical" waves that "travel" from the two Black Holes toward any region of the Universe (Earth included). On the contrary, they are understood as *Ordinal Topological* variations of the *Proper Spaces* of the two Black Holes, as a consequence of their *Generative Ordinal Inter-Action*.

Such a Generative Process, in turn, cannot be considered as being "limited" to the two sole Black Holes. In fact, in the context of an Ordinal Description, the two Black Holes cannot be "isolated" as sort of a "*sub-System*" of the entire Universe. This is because, as we have seen at par. 2.2, in the case of non-living Systems we *never have a possible distinction between System and sub-Systems*.

Consequently, the two Black Holes have to be seen (at least) in the context of a very wide "portion" of the Universe (including Earth), which has always to be understood as Self-Organizing System and thus characterized by *a unique and sole* Specific Generativity. Such a "portion" of the Universe, precisely because understood as a Self-Organizing System, will always tend toward its Maximum Ordinality, even in the presence of the strong Ordinal variations generated by the Ordinal Inter-Action between the two Black Holes.

The two Fundamental Equations of the MOP are still able to describe such a Generative Evolution, although in the presence of the new "origin and habitat conditions".

The corresponding "Emerging Solution" will then show the Generative Evolution of such a wider System, always understood as a Whole, while it is tending toward a "new" level of Maximum Ordinality. In such a context it is possible to recognize the specific Ordinal "Reflex" of the corresponding "Ordinal Variations" of the Proper Space and Proper Time of the two Black Holes. As well as the specific "Reflex" of the corresponding "Ordinal Variations" of the Proper Space and Proper Time of the instrumentation on Earth which, however, are related to the former *only* in terms of *new Harmony Relationships*.

This consequently means that the "phenomenon" revealed by the instrumentation on Earth is *not directly referable* to some *efficient and functional causes* (such as the supposed "physical" waves). On the contrary, it is the direct "Reflex" of the new Ordinal Topological Reconfiguration of the abovementioned *"portion" of the Universe*, in the presence of the new "origin and habitat conditions", directly related to the Ordinal Inter-Action between the two Black Holes.

CONCLUSIONS

The major novelty presented in this paper is that the MOP, finalized to describe the "Emerging Quality" of Self-Organizing Systems, is always valid for both *non-living* Systems, *Living* Systems and *Human* Systems, and it is also able to "offer" their corresponding "Emerging Solutions" always in an *explicit form*.

In addition, the same "Emerging Solutions" so obtained reveal another important novelty: each Self-Organizing System evolves in its "*Proper Time*" and "*Proper Space*", which are strictly specific of the System analyzed.

Such these two new gnoseological concepts enabled us to reinterpret in a different (although always com-possible) perspective the Einstein *space-time conception* and, consequently, the recent *related discovery* of the "gravitational waves".

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www.ordinality.org: author's website that presents a general framework about the MOP, by starting from the Mathematical Formulation of Odum's Maximum Em-Power Principle up to the Mathematical Formulation of the MOP, together with some Ostensive Examples mentioned in this paper.